Method of extreme points: forecast of population dynamics (on an example of *Zeiraphera diniana* Gn.)

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Abstract

In current publication method of extreme points is applied for forecast of larch bud moth population (*Zeiraphera diniana* Gn.) dynamics (GPDD 1407). For Moran – Ricker model feasible sets (in a space of model parameters) were determined for part of time series, and last parts of time series were used for checking of properties of forecast. For *tails* of time series and for all model trajectories (with parameters from feasible sets) maximum and minimum forecasting values of population size were found. For forecast average values and some selected trajectories with extreme properties were used. Forecasting properties of pointed out values are under discussion.

Keywords larch bud moth population dynamics, time series, statistical analysis, Moran-Ricker model, fitting, forecast, method of extreme points

Introduction

Search of suitable mathematical model and estimation of model parameters using empirical datasets are among of main elements of population dynamics analysis (McCallum, 2000; Isaev et al., 1984, 2001; Turchin, 2003; Wood, 2001; Nedorezov, Lohr, Sadykova, 2008; Nedorezov, Utyupin, 2011). Without finding of suitable model (or without constructing of new suitable model) it is impossible to prepare scientifically based forecasts of population size changing, optimal methods of its management etc. But up to current moment there are no criterions, which can help in finding suitable model before comparison of theoretical and empirical results (Isaev et al., 1984, 2001). In such a situation various methods of preliminary statistical analysis, which can help in creation of plausible hypothesis about the character of population fluctuations, can play important, key role in choosing of mathematical models (Nedorezov, 2012, 2014).

As it was proved in our previous publications (Nedorezov, 2011; Sadykova, Nedorezov, 2013; Nedorezov, Sadykova, 2015) it is possible to obtain good fitting of empirical time series of larch bud moth (*Zeiraphera diniana* Gn.) population (Auer, 1977; Baltensweiler, Fischlin, 1988) using Moran – Ricker model (Moran, 1950; Ricker, 1954):

$$x_{k+1} = Ax_k e^{-\alpha x_k} . \quad (1)$$

In (1) x_k is population density (or population size) at moment k; parameter A is a maximum birth rate, and α is a coefficient of self-regulation. This model has very rich set of dynamic regimes, and it is determined very wide application of this model to description of dynamics of various populations (McCallum, 2000; Nedorezov, 1986, 1997; Nedorezov, Nedorezova, 1994; Turchin, 2003 and many others).

In current publication we use model (1) in the following manner. For part of time series (21 first elements of initial sample or more) we determine points of feasible set (for 5% and 20% significance levels). After that we use points from feasible sets for determination of various forecasting characteristics (for other elements of sample): maximum and minimum values of population density, and average values. Trajectories with some extreme properties (with minimum value of characteristics of Kolmogorov – Smirnov statistical test) were also used for forecast.

Datasets

Regular observations of larch bud moth fluctuations had been started in Swiss Alps (in Upper Engadine Valley) in 1949 (Auer, 1977; Baltensweiler, Fischlin, 1988). Used in current publication time series can be free downloaded in Internet (NERC Centre for Population Biology, Imperial College (1999) The Global Population Dynamics Database, N 1407). Population densities are presented in units “number of larvae per kilogram of branches”. Data were collected in Upper Engadine Valley on 1800 m above the sea level. Total sample size is 38 values (from 1949 to 1986).

Statistical tests

Let $\{x_k^*\}$, $k = 0,1,\ldots,N$, be empirical time series of population density changing in time; $N+1$ is sample size. Using this sample $\{x_k^*\}$ we have to estimate model parameters A, α , and initial population size x_0.

Use of least square method (Bard, 1974; Borovkov, 1984) is based on assumption that best estimations of model parameters can be found with minimizing of sum of squared deviations between theoretical (model) and empirical datasets. If time series is approximated by model trajectory (1) loss-function can be presented in the following form:

$$Q(\vec{\alpha}, x_0) = \sum_{k=0}^{N} \left(x_k(\vec{\alpha}, x_0) - x_k^* \right)^2 . \quad (2)$$

In (2) $\{x_k(\vec{\alpha}, x_0)\}$ is model (1) trajectory obtained for fixed values of vector $\vec{\alpha} = \{A, \alpha\}$ and x_0. Let's also assume that for certain point $(\vec{\alpha}^{**}, x_0^{**})$ there is a global minimum in (2):

$$Q(\vec{\alpha}^{**}, x_0^{**}) = \min_{\vec{\alpha}, x_0} \left(\sum_{k=0}^{N} \left(x_k(\vec{\alpha}, x_0) - x_k^* \right)^2 \right). \quad (3)$$

Following a traditional approach of solution of considering problem (Bard, 1974; Borovkov, 1984; Draper, Smith, 1981) after determination of estimations $(\vec{\alpha}^{**}, x_0^{**})$ (3) analysis of set of deviations $\{e_k\}$ between theoretical and empirical datasets must be provided:

$$e_k = x_k(\vec{\alpha}^{**}, x_0^{**}) - x_k^* . \quad (4)$$

Model (1) is recognized to be suitable for fitting of considering time series if following conditions are truthful: deviations $\{e_k\}$ (4) are values of independent stochastic variables with Normal distribution and zero average. Following these assumptions Kolmogorov – Smirnov, Lilliefors, Shapiro – Wilk or other tests are used for checking of Normality of deviations (Bolshev, Smirnov, 1983; Lilliefors, 1967; Shapiro, Wilk, Chen, 1968). For checking of independence of stochastic variables Durbin – Watson and/or Swed – Eisenhart tests are used (Draper, Smith, 1981; Hollander, Wolfe, 1973; Likes, Laga, 1985).

If in the sequence of residuals (4) serial correlation is observed it gives a background for conclusion that considering model isn't suitable for fitting and needs in modification. It means also that some of important factors or processes were not taken into account within the framework of model. Similar conclusion about model and its applicability to fitting can be made in situation when hypothesis about Normality of deviations must be rejected (for selected significance level). In other words, final conclusion about suitability of model for approximation of considering time series is based on analysis of properties of unique point $(\vec{\alpha}^{**}, x_0^{**})$ in the space of model parameters.

In our opinion, this is one of basic problems of least squared method: a'priori it is impossible to exclude from consideration a situation when nearest to $(\vec{\alpha}^{**}, x_0^{**})$ points have required properties. Below we'll consider situations when deviations between theoretical and empirical datasets are satisfied to following conditions. First of all, distribution of deviations (4) must be symmetric with respect to origin. Branches of density function must be monotonic curves – it must increase in negative part of straight line, and it must decrease in right part.

Hypotheses about existence of serial correlation in sequences of residuals must be rejected (for selected significance level).

For checking of these properties of residuals we used Kolmogorov – Smirnov test, Lehmann – Rosenblatt test, and Mann – Whitney test ((for checking of symmetry of distribution; Bolshev, Smirov, 1983; Hollander, Wolfe, 1973; Likes, Laga, 1985). For checking of monotonic behavior of branches of density function Spearman rank correlation coefficient was used. For testing of absence of serial correlation Swed – Eisenhart test (Draper, Smith, 1981) and test “jump up –jump down” (Likes, Laga, 1985) were used.

Point of space of model parameters (x_0, A, α) belongs to feasible set if and only if for selected significance level all statistical tests show requirement results: hypotheses about symmetry cannot be rejected, hypothesis about equivalence of Spearman rank correlation coefficient ρ to zero must be rejected (with alternative hypothesis $\rho > 0$), hypotheses about existence of serial correlation must be rejected. It is obvious that geometry of feasible set depends on statistical criterions we use for checking of properties of deviations and on selected significance level. Note, that for various tests we can use different significance levels.

In current publication for every fixed part of time series (21 first elements or more) for 5% feasible sets were determined. For all points of feasible set (respectively, for all trajectories of model (1)) maximum values, minimum values and averages were determined. For constructing of forecast trajectory of model (1) with minimum value of Kolmogorov – Smirnov test was used too.

As it was obtained, feasible sets with 5% significance level are very wide and obtained results are far from observed values. For obtaining better forecasting results for some tests (Kolmogorov – Smirnov test, Lehmann – Rosenblatt test, Swed – Eisenhart test, and test “jump up –jump down” were increased up to 20%. For narrower feasible sets same forecasting characteristics were determined.

Results

Stochastic points (x_0, A, α) were determined in set $[0,1000]\times[0,1000]\times[0,100]$ with uniform distribution. First two limits were determined from condition that observed values in two times less than pointed out amounts. Maximum of population density is 450, maximum value of birth rate is 333.33. It was additionally assumed that number of positive and negative deviations (4) cannot be less than 40% of considering sample.

First sample contained 21 points of initial sample (values from 1949 and up to 1969). Respectively, in this case forecast was prepared for time interval 1970-1986. If it was checked that point belongs to feasible set (all statistical tests were satisfied), for these parameter trajectory of model (1) was determined, and obtained values were used for maximum, minimum and average forecasting values. All these values were determined for all 150000 stochastic trajectories (for 150000 elements of feasible sets).

Results of calculations for first case are presented in figure 1. Minimum values don't presented in this figure: all obtained values are rather small and less than 10^{-129}. Curve 3 corresponds to case when characteristics of Kolmogorov – Smirnov test has its minimum value (0.29129; probability that distribution of deviations isn't symmetric is less than 10^{-5}; it means that hypothesis about symmetry *must be accepted*).

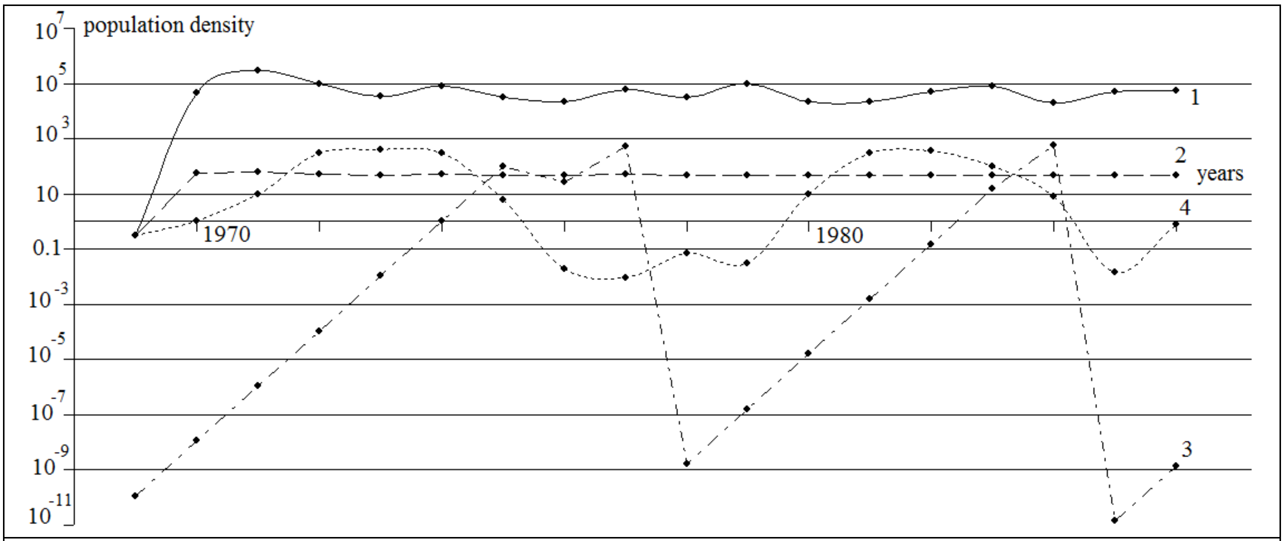

Fig. 1. Forecast of changing of larch bud moth dynamics for time interval 1970-1986. Curve 1 is maximum forecasting values of population density. Curve 2 contains average values of population densities. Curve 3 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 4 is empirical time series. Results are presented in logarithmic scale.

In figure 2 the same datasets (without maximum forecasting values) are presented in arithmetic scale. As we can see in these figures, average values change in rather small diapason. Thus, use of these characteristics for forecast of mass propagations looks rather problematic. Use of curve 3 looks problematic too: maximum values of this curve are observed after several years (fig. 1 and 2) of outbreak of larch bud moth.

It is important to note that correlation coefficient between real trajectory (1970-1986) and sequence of maximum values is equal to -0.07842; correlation coefficient between real trajectory and sequence of minimum values is equal to -0.17803; correlation coefficient between real trajectory and sequence of averages is equal to -0.20589. Finally, correlation coefficient between real trajectory and trajectory 3 is equal to -0.27642. It allows concluding that used characteristics cannot be used for forecast. But observed not good correspondence between used characteristics and tail of time series may depend on selected significance level, selected characteristics (we cannot exclude a situation when extreme trajectory with minimum functional (2) on feasible set will demonstrate good forecast) etc.

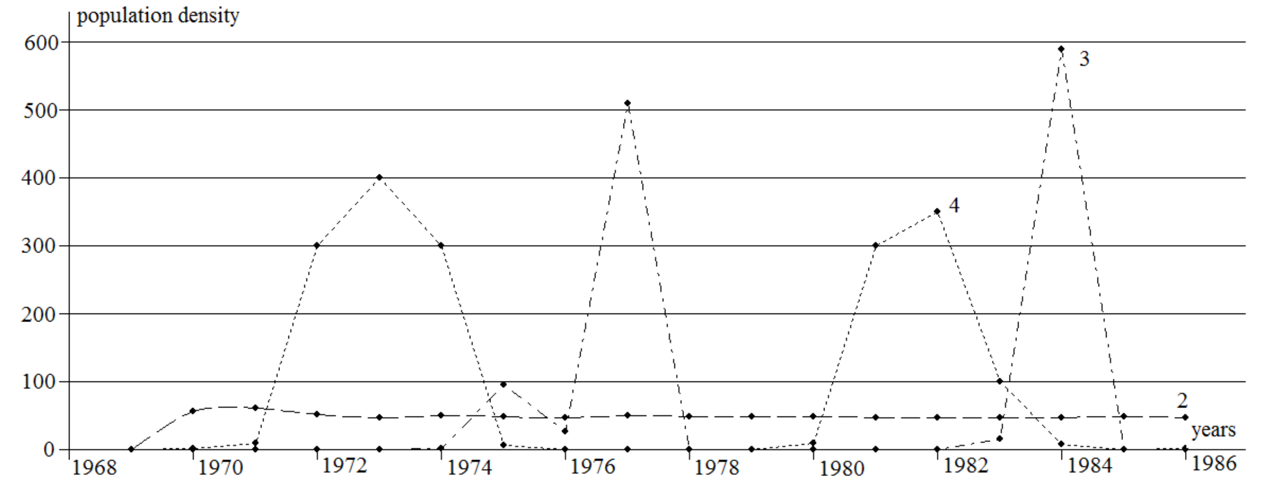

Fig. 2. Forecast of changing of larch bud moth dynamics for time interval 1970-1986. Curve 2 contains average values of population densities. Curve 3 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 4 is empirical time series. Results are presented in arithmetic scale.

Similar behavior of curves was observed in other cases. For example, in figures 3 and 4 results of calculations for situation when for determination of feasible set 22 first elements of initial sample were used, are presented.

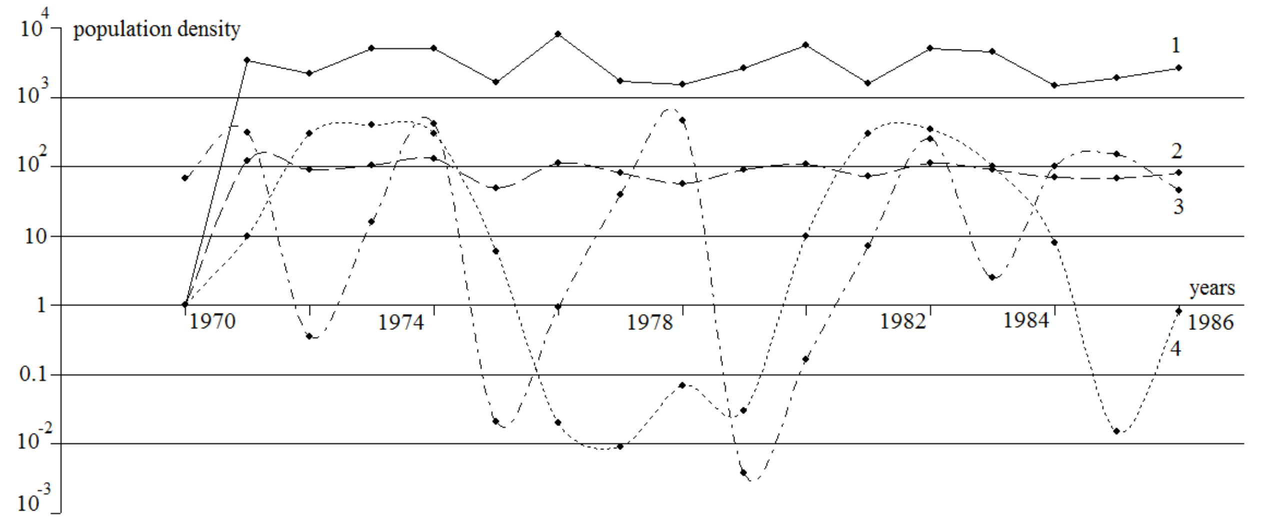

Fig. 3. Forecast of changing of larch bud moth dynamics for time interval 1971-1986. Curve 1 is maximum forecasting values of population density. Curve 2 contains average values of population densities. Curve 3 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 4 is empirical time series. Results are presented in logarithmic scale.

Correlation coefficient between real trajectory (1971-1986) and sequence of maximum values is equal to 0.215725; correlation coefficient between real trajectory and sequence of minimum values is equal to -0.18991; correlation coefficient between real trajectory and sequence of averages is equal to 0.39033. Finally, correlation coefficient between real trajectory and trajectory 3 is equal to -0.059764. It allows also concluding that used characteristics cannot be used for forecast. Best result was obtained for sequence of averages. But in all cases we got rather small values of correlation coefficients.

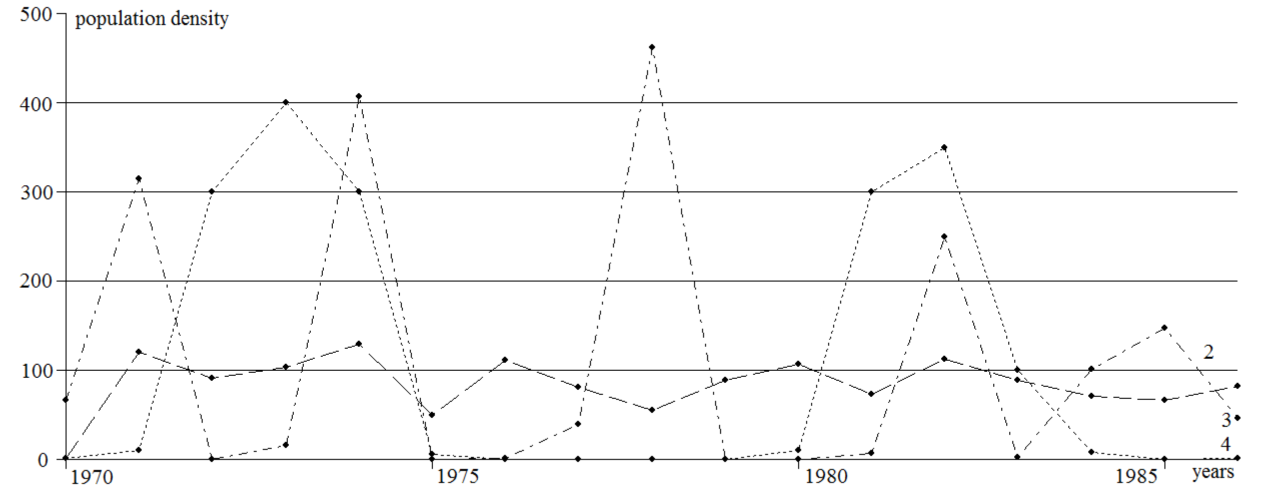

Fig. 4. Forecast of changing of larch bud moth dynamics for time interval 1971-1986. Curve 2 contains average values of population densities. Curve 3 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 4 is empirical time series. Results are presented in arithmetic scale.

Note, that with 5% significance level maximum and minimum forecasting limits (fig. 1 and 2) are without of practical interests: there are very big differences between forecasting limits and real datasets. At the same time extremum value of upper boundary is observed in 1971 (respective maximum of time series is observed in 1973); next extremum (with respect to its amount) is observed for upper limit in 1979 (respective maximum of time series is observed in 1982). Thus, in two-three years before phase of maximum of population outbreak we can observe extreme values of upper limit; moreover, bigger value of upper limit corresponds to bigger value of population density (fig. 1 and 2).

Similar behavior of considering characteristics, similar properties of relations between real dataset and characteristics were observed for other cases when part of initial sample contains 23 or more elements. This situation was changed after selection of other amount of significance level for some of used statistical tests. When requirements to sets of deviations become stronger it leads to decreasing of size of feasible sets and, as a result of pure stochastic search, to appearance of trajectories with close properties.

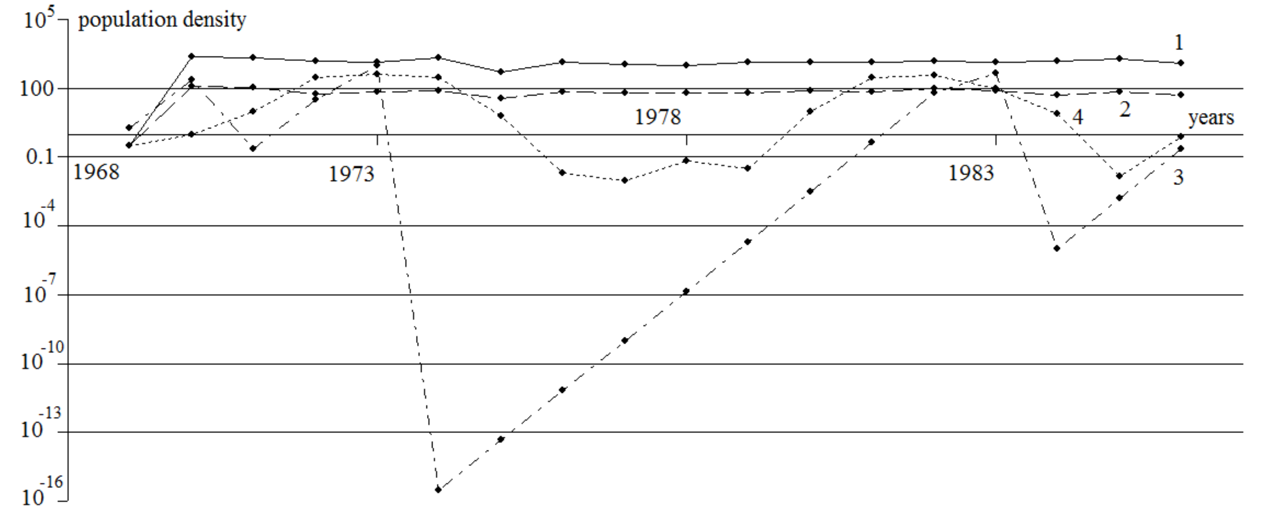

Fig. 5. Forecast of changing of larch bud moth dynamics for time interval 1970-1986 (after increasing of value of significance level for some statistical tests). Curve 1 is maximum forecasting values of population density. Curve 2 contains average values of population densities. Curve 3 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 4 is empirical time series. Results are presented in logarithmic scale.

It was assumed that hypothesis about symmetry of distribution cannot be rejected by Kolmogorov – Smirnov test and Lehmann – Rosenblatt test with 20% significance level. It was also assumed that with 20% significance level hypothesis about absence of serial correlation cannot be rejected by Swed – Eisenhart test, and test “jump up –jump down”. Results of calculations are presented in figures 5 and 6.

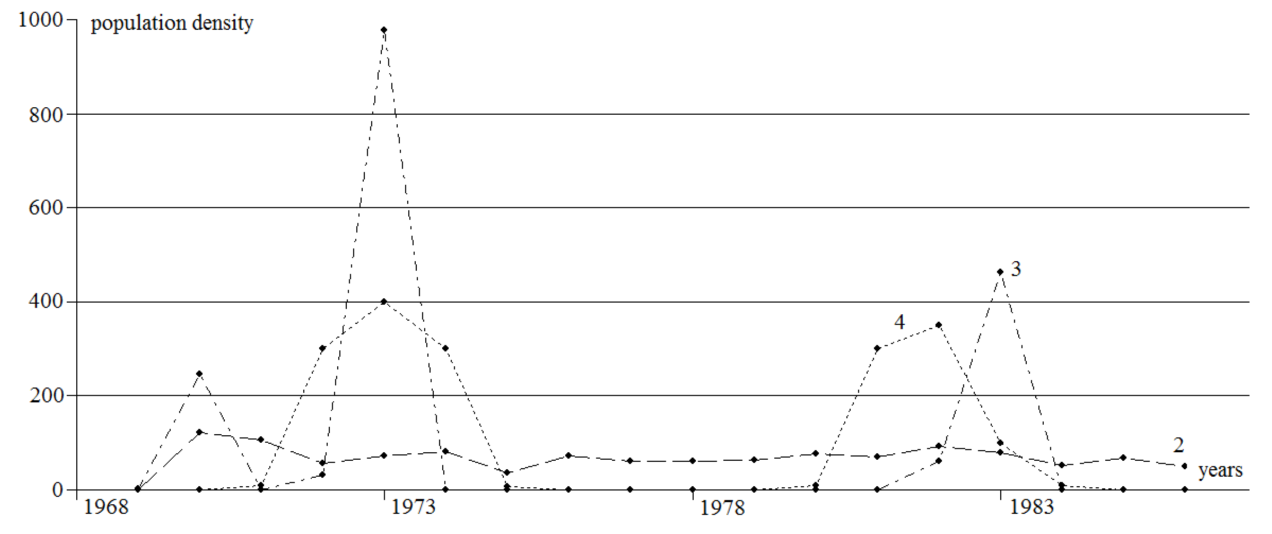

Fig. 6. Forecast of changing of larch bud moth dynamics for time interval 1970-1986 (after increasing of value of significance level for some statistical tests). Curve 2 contains average values of population densities. Curve 3 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 4 is empirical time series. Results are presented in arithmetic scale.

As we can see in these figures, interesting behavior demonstrates by curve 3: before population outbreak (in 1973) it has local maximum, and it has global maximum in 1973. Next maximum it has in 1983, and it is after outbreak in 1981-1982 (fig. 6). Maximum forecasting limits decreased in 13.95 times as a minimum; in one case it was decreased in 137.15 times. But in all cases values of upper limit is bigger than values of initial sample. Low limits were increased, and forecasting diapason became narrower. At the same time all values of low limits are close to zero and less than 10^{-38}.

For considering situation correlation coefficient between real trajectory (1971-1986) and sequence of maximum values is equal to 0.120266; correlation coefficient between real trajectory and sequence of minimum values is equal to -0.16028; correlation coefficient between real trajectory and sequence of averages is equal to 0.104521. Finally, correlation coefficient between real trajectory and trajectory 3 is equal to 0.448498. It allows concluding that used three first characteristics cannot be used for forecast (correlation coefficients are very small). Best result was obtained for trajectory 3 (fig. 6), and for this trajectory correlation coefficient between real time series and trajectory has maximum value.

Conclusion

Constructing of feasible sets in space of model parameters allows obtaining and using for forecast various characteristics of these sets. In particular, it allows using trajectories with extreme properties (of sets of deviations between theoretical and empirical datasets). In ideal situation we have one trajectory only which has extreme properties for deviations. But such situations are very rare. In most cases we can find trajectories which have, for example, symmetric distribution for deviations with a certain guarantee (in a case when hypothesis about symmetry cannot be rejected with 99.999% or more significance level). It is also possible to use least squared method within the limits of feasible sets etc. But up to current moment it isn't obvious what kind of characteristics of feasible sets we have to use for constructing best forecasts.

Results obtaining for feasible sets with 5% significance level, show that feasible sets are very big, and in a result of it maximum and minimum forecasting limits are very far from observed values. It means that these amounts cannot be used for forecast and, additionally, for obtaining more realistic results we have to use narrower feasible sets (in particular, with 20% significance level for some statistical tests).

References

Auer C. 1977. Dynamik von Larchenwicklerpopulationen langs des Alpenbogens. Mitteilungen der Eidgenossischen Anstalt fiir forstliches Versuchswesen 53: 71-105.

Baltensweiler W., Fischlin A. 1988. The larch budmoth in the Alps// Berryman A.A. (ed): Dynamics of forest insect populations: patterns, causes, implications. Plenum Press: New York: 331-351.

Bard Y., 1974. Nonlinear Parameter Estimation. Academic Press Inc., New York.

Bolshev L.N., Smirnov N.V., 1983. Tables of Mathematical Statistics. Nauka, Moscow.

Borovkov A.A., 1984. Mathematical Statistics. Nauka, Moscow.

Draper N.R., Smith H., 1981. Applied Regression Analysis. Wiley and Sons Inc., New York.

Hollander M.D.A., Wolfe D.A., 1973. Nonparametric statistical methods. New York, London, Sydney, Toronto. John Wiley and Sons, Inc..

Isaev A.S., Khlebopros R.G., Nedorezov L.V., Kondakov Yu.P., Kiselev V.V. 1984. Forest insect Population Dynamics. Nauka, Novosibirsk.

Isaev A.S., Khlebopros R.G., Nedorezov L.V., Kondakov Yu.P., Kiselev V.V., Soukhovolsky V.G. 2001. Population Dynamics of Forest Insects. Nauka, Moscow.

Likes J., Laga J., 1985. Basic tables of mathematical statistics. Finance and statistics, Moscow.

Lilliefors H.W., 1967. On the Kolmogorov–Smirnov test for normality with mean and variance unknown. J. Amer. Statist. Association 64: 399-402.

McCallum H. 2000. Population parameters estimation for ecological models. Brisbane: Blackwell Sciences Ltd.

Moran P.A.P. 1950. Some remarks on animal population dynamics. Biometrics 6(3): 250-258.

Nedorezov L.V. 1986. Modeling of Forest Insect Outbreaks. Nauka, Novosibirsk.

Nedorezov L.V. 1997. Course of Lectures on Ecological Modeling. Siberian Chronograph, Novosibirsk.

Nedorezov L.V. 2011. Analysis of Cyclic Fluctuations in Larch Bud Moth Populations with Discrete-Time Dynamic Models. Biology Bulletin Reviews 72(2): 407-414.

Nedorezov L.V., 2012. Chaos and Order in Population Dynamics: Modeling, Analysis, Forecast. LAP Lambert Academic Publishing, Saarbrucken.

Nedorezov L.V. 2014. Use of Moran-Ricker model for fitting of larch bud moth time series: structure of feasible set in space of model parameters. Population Dynamics: Analysis, Modelling, Forecast 3(4): 102-109.

Nedorezov L.V., Lohr B.L., Sadykova D.L. 2008. Assessing the importance of self-regulating mechanisms in diamondback moth population dynamics: Application of discrete mathematical models. J. of Theoretical Biology 254: 587–593.

Nedorezov L.V., Nedorezova B.N. 1994. Modifications of Moran-Ricker models of isolated population dynamics. Journal of General Biology, 55(4-5): 514-521

Nedorezov L.V., Sadykova D.L. 2015. Dynamics of larch bud moth populations: application of Moran - Ricker models with time lag. Ecological Modelling 297: 26-32.

Nedorezov L.V., Utyupin Yu.V. 2011. Continuous-Discrete Models of Population Dynamics: An Analytical Overview. State Public Scientific-Technical Library, Novosibirsk.

Ricker W.E. 1954. Stock and recruitment. J. Fish. Res. board of Canada 11(5): 559-623.

Sadykova D.L., Nedorezov L.V. 2013. Larch bud moth dynamics: can we explain periodicity of population fluctuations by the time lag-dependence in birth rate? Population Dynamics: Analysis, Modelling, Forecast 2(4): 154-181.

Shapiro S.S., Wilk M.B., Chen H.J. 1968. A comparative study of various tests of normality. J. of the American Statistical Association 63: 1343-1372.

Turchin P. 2003. Complex Population Dynamics: A Theoretical/Empirical Synthesis. Princeton University Press, Princeton, USA

Wood S.N. 2001. Partially specified ecological models. Ecological Monographs, 71: 1-25