Short Communication

Use of Moran-Ricker model for fitting of larch bud moth time series: structure of feasible set in space of model parameters

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For estimation of parameters of nonlinear dynamic models one can use various statistical methods. In particular, it can be least square method when we minimize squared deviations between empirical and theoretical values (Bard, 1974; Bolshev, Smirnov, 1983; Draper, Smith, 1998 and many others). After finding of parameter estimations when minimizing functional has global minimum, sequence of deviations is analyzed by a certain number of statistical tests. It can include tests on Normality with zero average (Lilliefors, 1967; Shapiro, Wilk, Chen, 1968), and tests on absence of serial correlation (Draper, Smith, 1998).

If serial correlation is observed in a sequence of residuals we have a background for conclusion that model isn't suitable for fitting of considering time series. The same conclusion we can get in a situation when distribution of residuals isn't Normal (for fixed significance level). It means that final conclusion about suitability of model for fitting of time series we make using *one point from the space of model parameters*. This point gives minimum for minimizing functional form. But what is a background for assumption that estimations of model parameters must give minimum for (any) functional form? The answer is rather obvious: this assumption has no background.

One more assumption about Normality of deviations has no background too. Moreover, in all real situations this assumption doesn't correspond to reality. For example, if we estimating weights of insects obtained dataset cannot correspond to Normal distribution: we'll never have insect with negative weight, we cannot have error in several tons of kilograms. But if we postulate that errors of measurements correspond to Normal distribution it means that a'priori we assume that we may have *insects with negative weight with positive probability*. With positive probability we may also have insects with weight of several tons.

Thus, within the framework of traditional approach there are several serious problems with assumptions we make a'priori, and with logic order of providing analysis. Before estimation of model parameters we have to formulate basic requirements to model and to deviations between theoretical and empirical values.

First of all, density function of deviations must be symmetric with respect to origin and have monotonic branches. Let $\{e_k^+\}$ be a set of positive deviations between theoretical and empirical values, $\{-e_k^-\}$ be a set of (positive) deviations when difference is negative. Testing of symmetry can be provided with following way: two samples $\{e_k^+\}$ and $\{-e_k^-\}$ must have one and the same distribution. In this situation we can use Kolmogorov – Smirnov test, Lehmann – Rosenblatt criterion, Mann – Whitney test etc. (Bolshev, Smirnov, 1983; Likesh, Laga, 1985; Hollander, Wolfe, 1973).

The second, monotonic decreasing of branches of density function can be checked using Spearmen correlation coefficient (Bolshev, Smirnov, 1983). The third, for testing on absence of serial correlation in sequence of residuals we can use Swed – Eisenhart test (Draper, Smith, 1998) and test on series of "jumps up" and "jumps down" (Likesh, Laga, 1985). Note, that all tests pointed out above are nonparametric.

Application for analysis of deviations all pointed out above tests cannot give us cogent argument for conclusion about suitability of model for fitting of empirical time series. We have no reasons to say that considering model is good if for every increasing intervals in time series model demonstrates decreasing and vice versa. Thus, we have to check hypothesis about quota of successive variants "increasing in time series – increasing in model" and "decreasing in time series – decreasing in model" among all observed situations. Let q be a quota of such situations. It is obvious, if model demonstrates good correspondence with empirical dataset, we have to reject Null hypothesis $H_0:\ q = 0.5$ (with alternative hypothesis $H_1:\ q = 0.5$).

Checking of properties of points of space of parameters (with finite steps of changing of values of parameters) will allow obtaining of *feasible set* for model where we can and have to find minimum of any minimizing functional. Note, in principle volume of feasible set can be used for comparison of various models from the stand point of their correspondence to considering empirical datasets.

Described above steps of analysis of sets of deviations between model and empirical time series were applied to well-known dataset on larch bud moth (*Zeiraphera diniana* Gn.) in Swiss Alps (NERC Centre for Population Biology, Imperial College (1999); the Global Population

Dynamics Database, N 1407; Baltensweiler, 1964, 1978). For fitting of dataset Moran – Ricker model (Moran, 1950; Ricker, 1954) was used:

$$x_{k+1} = ax_k e^{-bx_k}, \; a, b = const \geq 0. \tag{1}$$

In (1) a is maximal birth rate; b is coefficient of self-regulation.

On pictures presented below there are points of two colors on the plane (a,b) – red and green. If for fixed values of model parameters a and b at least one initial value of population density x_0 can be pointed out (when the respective trajectory satisfies all described above statistical tests), this point marked as red. If respective initial value x_0 cannot be found or some of tests cannot be applied (for example, when sample size is rather small) or values of model trajectory cannot be calculated the respective point was marked as green.

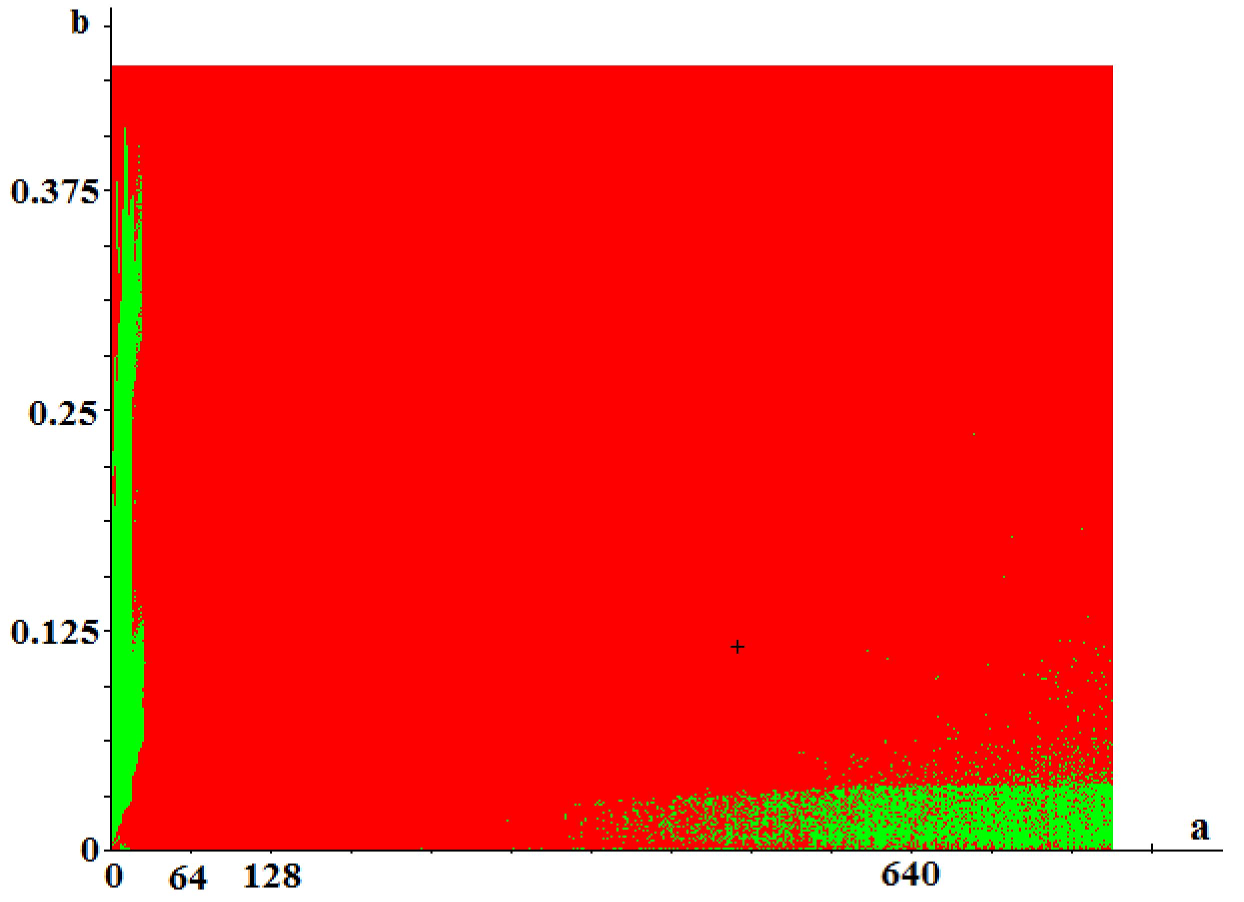

Fig. 1. Projection of feasible set on plane (a,b) after checking of first three tests: Kolmogorov – Smirnov test, Lehmann – Rosenblatt test, and Mann – Whitney test. Crest corresponds to point of global minimum of loss-function (squared deviations).

On figure 1 structure of feasible set after applying of first three tests (Kolmogorov – Smirnov, Lehmann – Rosenblatt, and Mann – Whitney tests) is presented (all pictures were prepared for 5% significance level). Crest corresponds to point of global minimum of loss-function which was calculated as squared deviations squared deviations between elements of initial empirical sample and respective elements of trajectory generated by model (1):

$$Q(a,b,x_0) = \sum_{k=0}^{n-1} \left(x_k(a,b,x_0) - x_k^* \right)^2 . \quad (2)$$

In (2) n is sample size ($n = 38$); $\{x_k(a,b,x_0)\}$, $k = 0,\ldots,37$, is trajectory generated by equation (1); $\{x_k^*\}$, $k = 0,\ldots,37$, is initial sample. Minimum of functional (2) $Q_{\min} = 465627.2$ was observed for $x_0 = 53.44$, $a = 500.62$, $b = 0.116$ (Nedorezov, 2011, 2012).

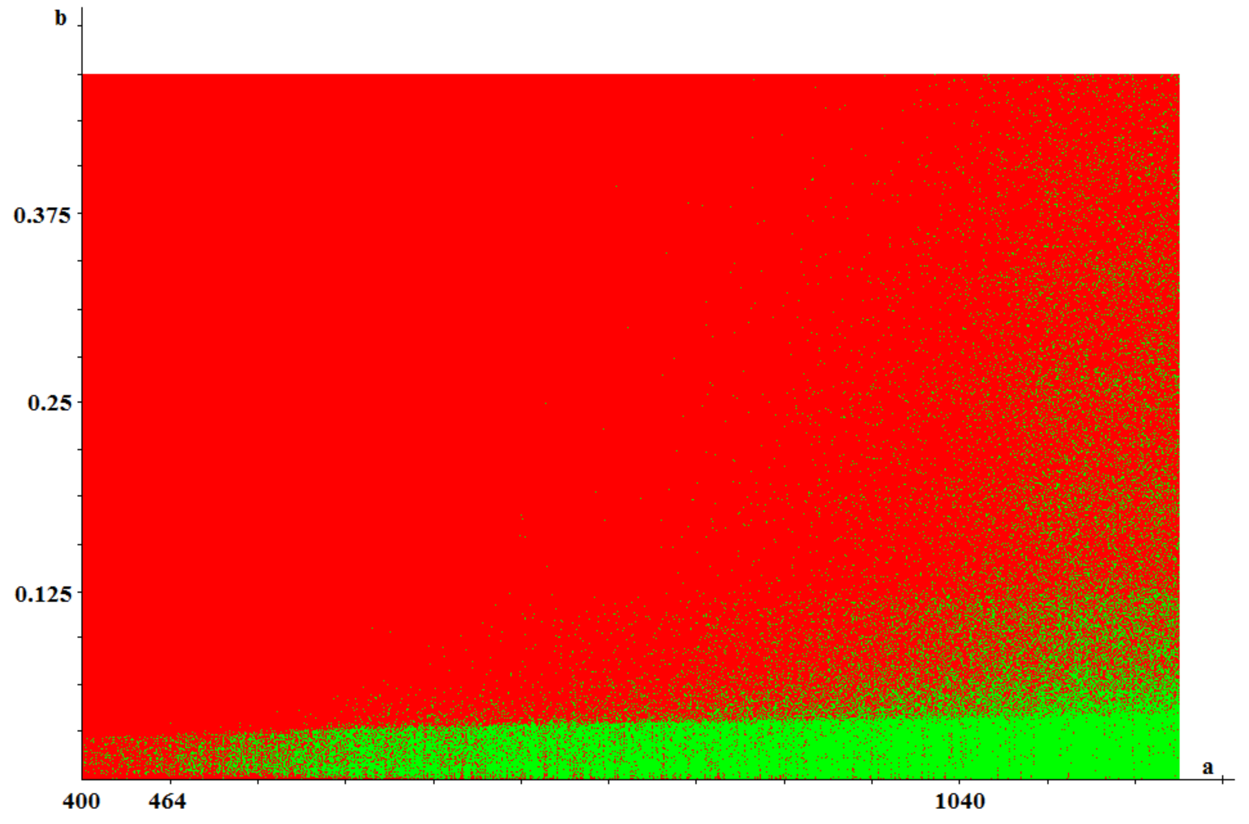

Fig. 2. Projection of feasible set on plane (a,b) after checking of first three tests: further increasing of parameter a.

Further increasing of value of parameter a (Fig. 2) leads to increasing of density of “green” points on the plane. On figure 3 there is the structure of feasible set in a situation when four statistical criterions were applied to testing of deviations (first three tests plus test on monotonic behaviour of branches of density function with Spearmen correlation coefficient; Bolshev, Smirnov, 1983). In considering variant (fig. 3) “red points” on the plane (a,b) correspond to situations when we can find at least one initial value x_0 when all applied criterions are satisfied. In other words, "red points" correspond to situations when we have no reasons to reject hypothesis about symmetry of density function with respect to origin and branches of this function are monotonic curves. It is obvious that feasible set became smaller than in previous considered case.

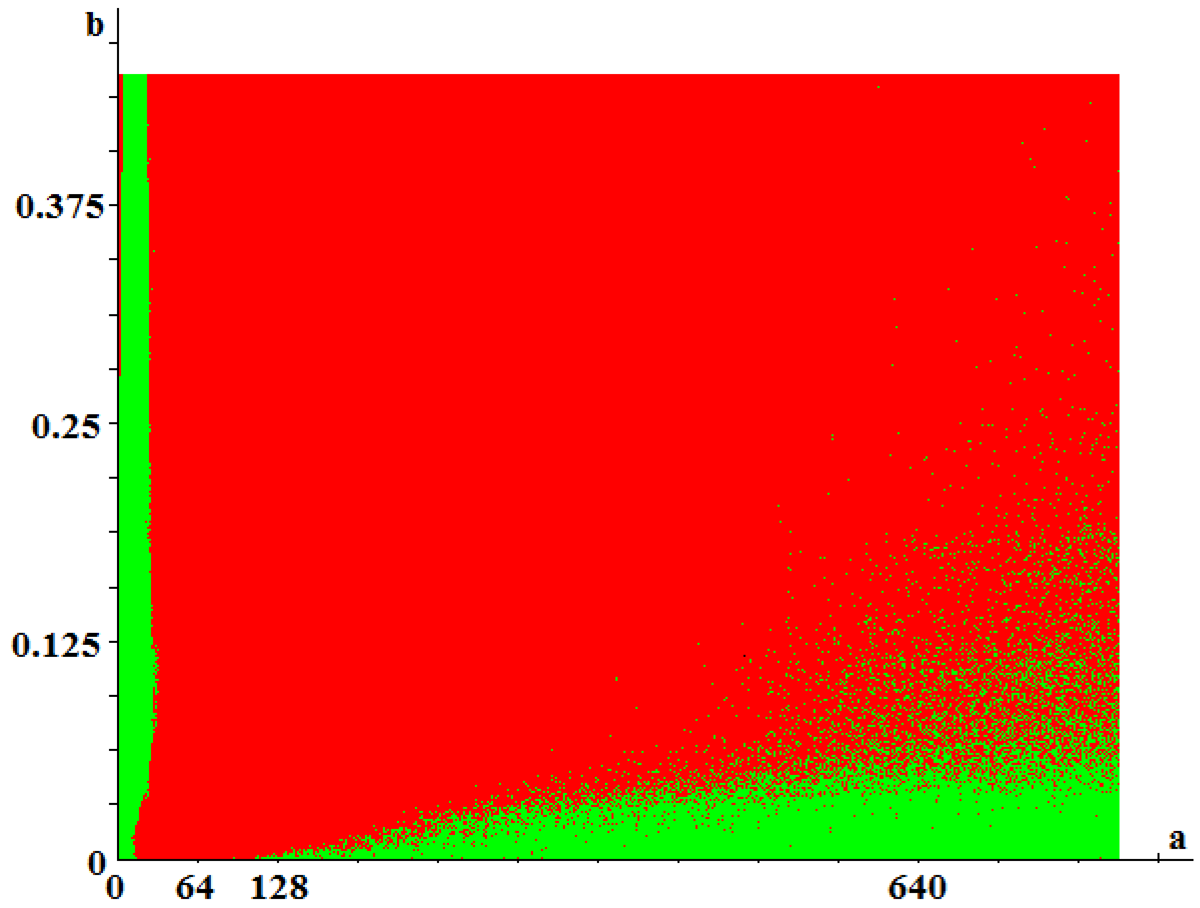

Fig. 3. Structure of feasible set after application of four various statistical criterions.

On the next step we have to check hypothesis about absence/existence of serial correlation in sequences of residuals. In many cases Durbin – Watson criterion is used for solution of this problem (Draper, Smith, 1998). But this test has two important problems. First of all, it is assumed that distribution of deviations must be Normal. But as it was pointed out above this assumption about Normality has no relation to biological tasks. And, the second, this test allows estimating of dependence in sequence of residuals with time lag in one year only.

Taking it into account for testing of dependence in sequence of residuals we used other tests: Swed – Eisenhart test (Draper, Smith, 1998) and test on series of "jumps up" and "jumps down" (Likesh, Laga, 1985). Results of application of six statistical tests are presented on figure 4. We have to note (fig. 4) that zone with "red points" is rather big. If we confine ourselves by the application of these statistical tests we have no reasons to say that Moran – Ricker model isn't suitable for fitting of time series on larch bud moth dynamics – we can point out values of model parameters when all used statistical criterions demonstrate good correspondence between theoretical (model) and empirical datasets.

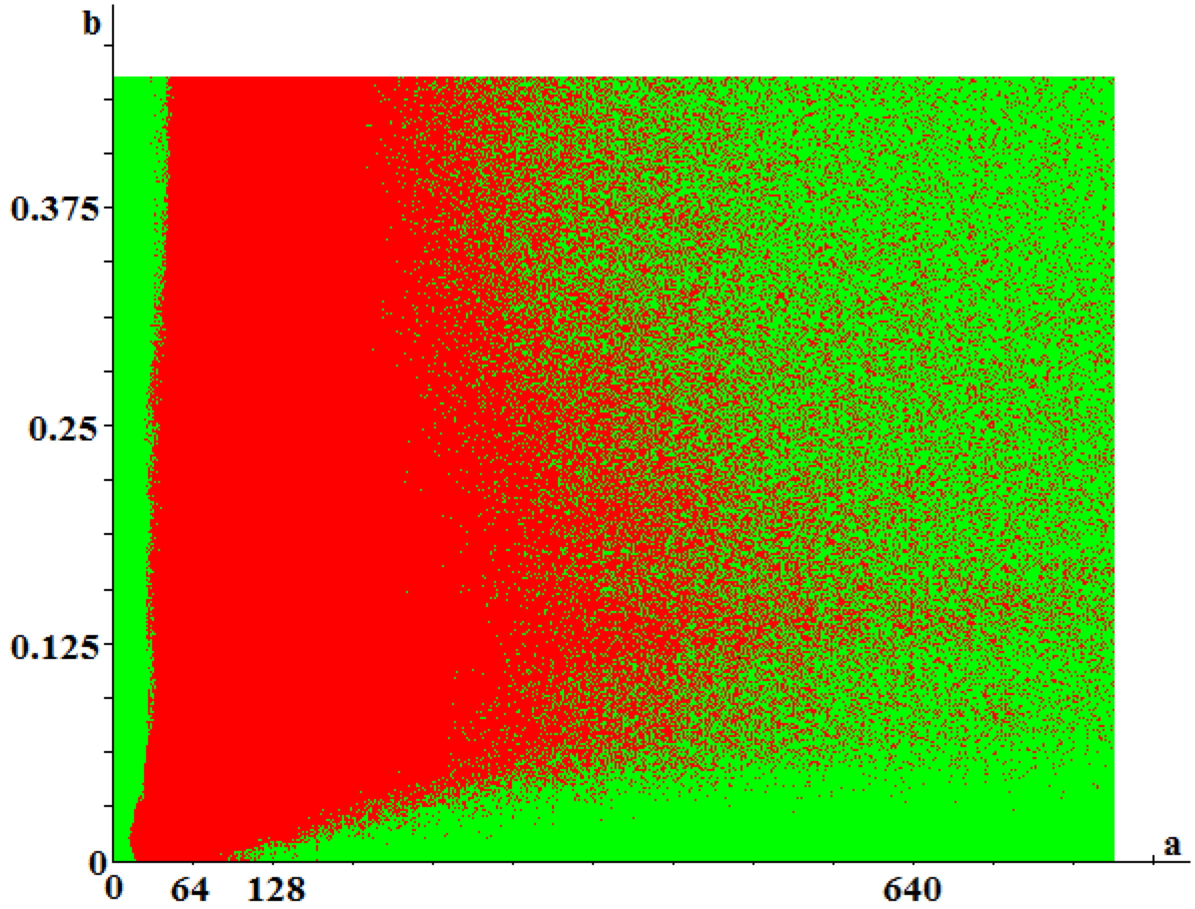

Fig. 4. Structure of feasible set after application of six various statistical criterions.

The last test – comparison of intervals of increasing and decreasing of model trajectory and time series, – isn't obligatory criterion. It can be omitted or modified in the following way. We haven't reasons to say that model is good and can be applied for fitting of considering time series if number of situations (quota of such situations q) when trajectory increases at moments of decreasing of time series and vice versa is rather big. It means that we can assume that Null hypothesis $H_0:\ q = 0.5$ must be rejected and accepted alternative hypothesis $H_1:\ q > 0.5$.

In current situation we assumed that Null hypothesis $H_0:\ q = 0.5$ must be rejected and accepted alternative hypothesis $H_1:\ q < 0.5$ if considering model is good and can be applied for fitting. Results of application of all statistical criterions are presented on figure 5.

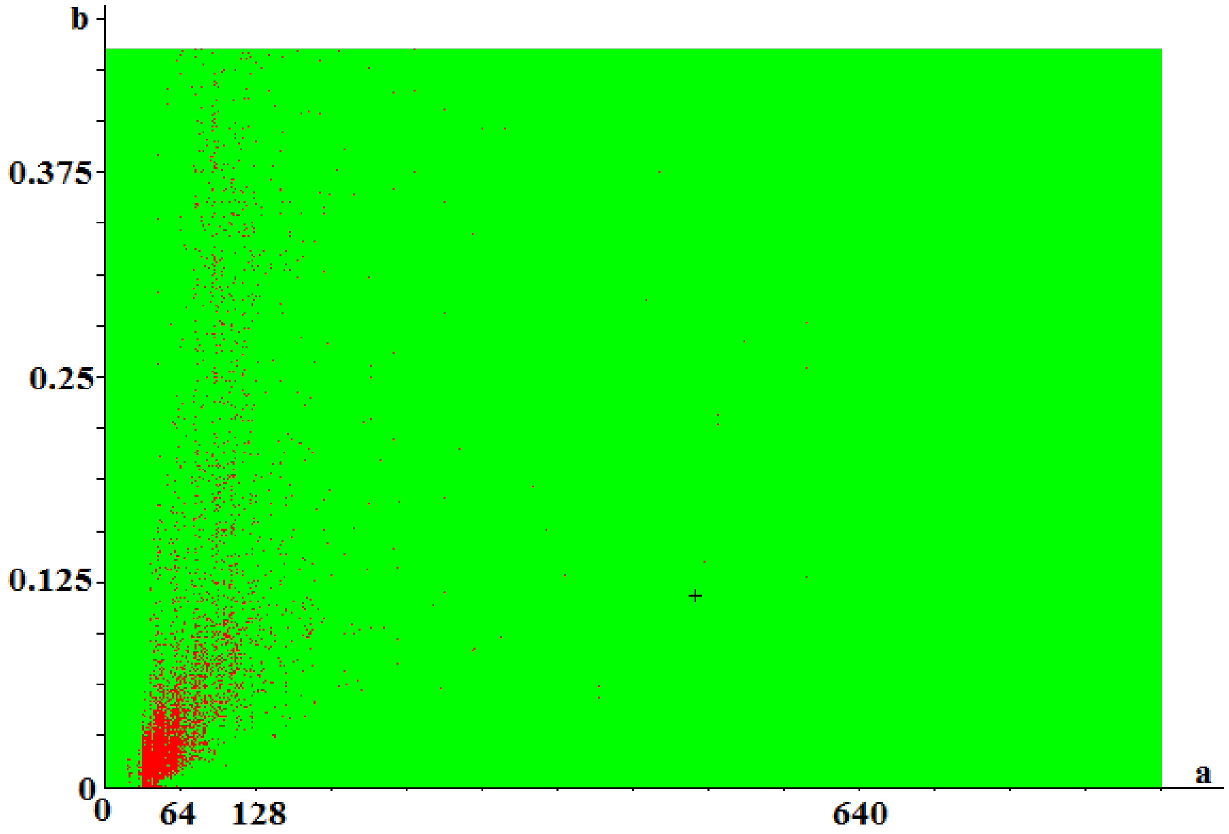

Fig. 5. Structure of feasible set after application of all statistical criterions.

As we can see on this picture (fig. 5) set of “red points” isn’t empty. It means that we can’t conclude that Moran – Ricker model cannot be applied for fitting of empirical time series. This result is in contradiction with result obtained with traditional approach. It is interesting also to note that global minimum of functional (2) is far from the set with highest concentration of “red points”.

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