Short Communication

Microtus arvalis Pall dynamics: reconsideration of an example from book by V.A. Kostitzin

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In book by V.A. Kostitzin (1937) the following example was presented for demonstration of situation when in optimum conditions fertility in population may remain constant for a long time (Chapter 4, page 57). For small rodent (*Microtus arvalis* Pall) data on local population dynamics and estimated with exponential function values were presented in table:

Table 1.

Months	0	2	4	6	8	10	12	14
$x(t)$ observed	2	5	16	20	40	109	200	283
$x(t)$ calculated	2	4.5	10	22	50	112	252	282

In table 1 $x(t)$ is population size at moment t. Calculations were provided with following formula:

$$x(t) = 2e^{0.4t}. \qquad (1)$$

In table 2 results of recalculation with formula (1) is presented. Results of fitting of observed values (table 1) with minimizing of squared deviations between exponential function and observed values allowed obtaining the next results: parameter of exponential function plus-minus standard error are equal to 0.360233±0.005318, value of loss function is equal to 4500.0472, $R^2 = 0.9418$. Loss function has the form:

$$Q(a) = \sum_{k=0}^{7} \left(x(k) - 2e^{2ak} \right)^2.$$

When parameter $a = 0.4$ value of loss function is equal to 68462.7. Note, it is much bigger than value of loss function when $a = 0.4$.

Traditional approach to analysis of deviations between theoretical and empirical datasets includes several steps: testing for Normality with zero average, and testing for existence/absence of serial correlation (Bard, 1974; Draper, Smith, 1998; Nedorezov, Utyupin, 2011; Nedorezov, 2012). For deviations average plus-minus standard error are equal to plus-minus standard error are equal to -10.466±9.428. Thus, with 5% significance level we have no reasons for rejecting hypothesis about equivalence of average to zero.

Table 2.

Months	0	2	4	6	8	10	12	14
$x(t)$ re-calculated	2	4.451	9.906	22.046	49.065	109.196	243.021	540.853
$x(t)$ calculated (new)	2	4.111	8.449	17.367	35.695	73.367	150.799	309.95

Kolmogorov – Smirnov test for Normality shows that $p > 0.2$; Lilliefors test gave the result $p < 0.15$; Shapiro – Wilk test gave the result $p = 0.48588$ (Bolshev, Smirnov, 1983; Lilliefors, 1967; Shapiro, Wilk, Chen, 1968). Thus, with 5% significance level we have no reasons for rejecting hypothesis about Normality of set of deviations.

Critical values for Durbin – Watson criteria are following (for 2.5% significance level): $d_L = 0.56422$, $d_U = 1.19119$. For obtained sample $d = 1.56353$. Thus, with 5% significance level we have no reasons for rejecting hypothesis about existence of serial correlation. For serial test (Draper, Smith, 1998) we have $n_1 = 6$, $n_2 = 1$, $u = 2$. Such a combination of signs of sample can be observed with rather big probability $p \approx 0.285714$.

Finally, conclusion which was made using nonoptimal value of parameter (Kostitzin, 1937), is truthful.

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