

SELF-ASSEMBLY OF MODELS: THEORY AND APPLICATION OF DESCRIPTION OF PROPERTIES OF COMPLEX SYSTEMS

Yu. P. Lankin

Krasnoyarsk Scientific Centre, Russian Academy of Sciences, Siberian Branch, Akademgorodok,
Krasnoyarsk, 660036 Russia

Siberian State Aerospace University, Krasnoyarsky Rabochy 31, Krasnoyarsk, 660014 Russia

e-mail: yuliylankin@gmail.com

Abstract

Fundamental problems of modern science are briefly analyzed that limit the development of classical and neoclassical methods for modeling and predicting the properties and dynamics of complex systems. This limitation prevents deep understanding of the fundamentals of functioning of natural and social systems and restricts the accuracy of predicting the behavior of these systems. It is proposed to overcome the mentioned difficulties by constructing the mathematics of self-assembly and self-initiation of models on the basis of deep qualitative generalizations of modern scientific advances. Successful research is grounded on the revelation and reflection of fundamental properties of complex natural systems in mathematical methods, in contrast to the attempts to squeeze these properties into the Procrustean bed of the representations based upon retrospective cognitive models. This young, intensively developed field of knowledge is named the concept of adaptive self-organization of complex systems.

Keywords: cognitive models, mathematical modeling, complex systems, self-assembly, adaptive self-organization, population dynamics, neural networks

Introduction

In previous studies (Lankin et al., 2012a–e), we considered some theoretical and application aspects of the concept of adaptive self-organization of complex systems as applied mainly to solving ecological problems. The concept, however, is universal, which becomes clearer as it is being arranged in the theory and the spectrum of interdisciplinary problems to be solved broadens. Among the important aspects in the development of the concept is the trend to

minimize interference of a researcher in the model formation. The effect of self-initiation of models is reached by using the methods of adaptive self-organization that are developed on the basis of the fundamental properties of complex natural systems studied with support of the latest achievements of synergetics (Haken, 2010), the fractal theory (Mandelbrot, 2004), the theory of chaos (Crownover, 1995), the theory of self-organized criticality (Bak, 1996), the theory of dynamic systems (Wiggins, 1997), the theory of complexity (Loskutov et al., 2007) and neuroinformatics (Haykin, 1998). In this study, the classical methodology of constructing mathematical models is briefly analyzed. Note that realization of the limitations imposed by classical and modern methods by no means denies the respect and sincere admiration for the scientists who created these methods and changed the world where their contemporary lived. Thanks to the great thinkers who were brave enough to oppose the truth they opened to dogmata of those times, even risking of losing authority, health, sometimes life, we can now move to new heights of knowledge.

Here, we demonstrate the possibility of building the mathematics of self-initiation of models on the basis of the concept of adaptive self-organization of complex systems.

System of equations: still not a model

When we write a system of equations, we do not construct a model yet. When equations are formed correct but coefficients are not fit, the system can behave anyhow yet not repeating the behavior of the object it was written for.

There are few variants of the behavior of an untuned model, depending on whether the positive or negative feedback (Wiener, 1965) prevails in the exchange by quantities between equations. If there is at least one positive feedback that breaks the equilibrium, the system passes either to the phase of the exponential growth of function values or, in case of delays in the exchange by quantities between the equations, to the oscillatory mode with an exponentially growing amplitude. In the presence of limitations, the system approaches them and the amplitude growth stops.

When there is the negative feedback, the system asymptotically converges to the stable state either in zero or at a point of the specified limitation that stops the amplitude drop.

Depending on a number of degrees of freedom and the quantity of synchronization of the system (Pikovskiy et al., 2003), the oscillatory modes can be more regular or more random.

It should be noted that classical methods are characterized by tendency to constructing simple models. However, practical experience of experts on modeling complex natural systems

can be expressed by phrase: “...simple models taking into account a small amount of parameters are inadequate and have no practical value” (Krapivin et al., 2002).

From the set of formulas and coefficients to the model

To transform the system of equations to the model, it is necessary to make identification or fitting (Schittkowski, 1994) of the model parameters or coefficients such as to the curves generated by the model pass through experimental data.

When the model is constructed, it operates as a model of a mechanism, following the rules specified once and for all. This is, however, inconsistent with the behavior of living systems, which continuously adapt to new conditions. Another problem is a small number (one to three) of stationary states obtainable in classical models. Meanwhile, it is known that living systems have numerous stationary states they acquire depending on current conditions. Nonliving systems also often have numerous potential wells and local minima, which ensure the preservation of the equilibrium states.

A serious difficulty of the classical methodology of model construction is the necessity of stepwise refinement of the latter. As the model becomes increasingly complex, which makes its behavior closer to the modeled system, the model development becomes more laborious and the probability of error in the mathematical description and computer code grows.

On the other hand, the use of classical modeling methods in constructing complex models leads to the well-known problems of (Krapivin et al., 2002) “the curse of dimensionality” and “the curse of instability” of a model. The first problem causes the rapid loss of the model comprehension and averaging of a great number of quantities, i.e., the loss of the required accuracy of a complex model. The second problem is related to the above-mentioned small number of the stationary states – dynamic equilibrium points – obtainable in classical models beyond which the model loses its stability. As the model becomes increasingly complex with large number of equations and coefficients, the number of its degrees of freedom increases and even a minor deviation from the stationary position shakes the solutions of many other equations. Thus, the behavior of the model represented by systems of equations is rapidly chaotic.

One more feature of complex models with a large number of equations and parameters is their ability of generating arbitrary curves or hypersurfaces at an appropriate selection of coefficients during identification. Thus, we arrive at the situation with numerous possible solutions, which is observed in reality. This situation obviously makes it impossible to speak about unambiguously separated mechanisms of functioning of complex systems.

It should be emphasized that the final goal of modeling the systems is study and prediction of their behavior with the use of the built models. One of the most effective methods

frequently used for this purpose is the qualitative analysis. It is perfect for two-dimensional models, but its application to three-dimensional models is complicated and, as the model dimensionality is further increased, the application difficulties grow avalanche-like. If a researcher still managed to accomplish the qualitative analysis of a multi-dimensional model, he often faces the problem of choosing from possible realistic variants of the model behavior.

All the aforesaid makes us speak about final destruction of the mechanistic model of the world, which has been implicitly kept in science since the Newton's times.

The transition from determinism to averaging of the statistical methods leads us to another problem of constructing the classical models. On one hand, averaging typical of any statistics smears the real complexity, which imparts such impressive capabilities to natural systems, and thus leads to the modeling accuracy loss. On the other hand, frequently used Gaussian distributions of an independent random variable weakly correlate with the structures of real systems represented by networks of interrelated objects, which are not isolated by definition. Isolation and independence of elements from one another denies the definition of a system as a complex of interrelated elements. Here, we again arrive at the question on adequacy of the mathematical methods. The use of the fat-tailed Pareto-Levy-type distributions (Pareto, 1964), which take into account the effects of interrelation of system elements, improves the situation only in case of the stationary processes, whereas in reality we have to deal, at best, with quasi-stationarity. Permanent variations in the degree of interrelation of elements of a system during its evolution lead to the instability of the statistical distributions, which frequently take place already at the stage of experiments and measurements (Mantegna et al., 2007). In addition, it must be noted that there exists a great number of statistical methods and distributions, apart from those mentioned above. This variety is caused by the absence of a general approach to the description of various systems and generates numerous particular cases during their modeling.

This brief excursus to the world's models originating in the times of perception of the ideal gas laws and Boltzmann statistics and averaging the world's diversity makes us draw sad conclusions similar to those made above for the mechanistic models of determinism.

Since the above-mentioned methods are effective only within narrow limits, what are those key ideas the mathematics reflecting the reality should base upon?

Will the neoclassical methods be salutary?

In the recent few decades, the science has been fortified by a number of impressive achievements, including synergetics (Haken, 2010) and the theories of catastrophes (Gilmor,

1993), dynamic systems (Wiggins, 1997), self-organized criticality (Bak, 1996), fractals (Mandelbrot, 2004), chaos (Crownover, 1995), etc.

Unfortunately, these amazing discoveries do not yield the final solution that would lead to the exact description of complex natural systems and high-quality predictions. Based on the classical concepts on “manual” model construction and strict regularities, they limit the representation of the diversity of the properties of complex systems without the loss of their self-identity or invariance. In particular, synergy and catastrophe theory allow forming a very small number of attractors or stationary states. The correlation of micro- and macro-levels is considered in the form of order parameters, i.e. mass trends in the behavior of microobjects. But the entire hierarchy and the elements at the micro-level are not reflected. Modeling of the systems is based upon reconstruction of individual attractors, without the concept of attractive landscape reconstruction. This leads to hypertrophied accenting on the “prediction horizon” obtained for simple models. Note that recently mathematical objects with a great number of stationary states (Anischenko, 2008) have been investigated without the ideas that would bring these ideal objects in correlation with the non-ideal configurations occurring during modeling of real complex systems. The fractal mathematics also demonstrates many interesting unusual objects, which seem realistic, but simultaneously assigns rigid iterative forms that do not reflect the diversity of the self-similarity properties of real objects in models.

Models of nature and observable reality

Most of classical and neoclassical models of complex systems are presented by systems of equations with a fixed form that are rigidly connected with one another by coefficients.

The situation is different in real natural systems. It especially concerns living systems. In a real system, not only values of connections between elements and subsystems, but the structure, configuration, and often properties of the system components described by model equations can vary during functioning. Therefore, the structure of a model claimed to fundamentally enhance the accuracy of predicting natural processes should continuously change, reflecting the variations in the structure of a modeled system. The structure of the model can transform and most of coefficients have no constant values and behave like slow variables, the values of which are modified via a feedback subsystem, such as to self-identity and invariance of a modeled system are permanently retained. All this extremely complicates the use of classical fitting or identification methods (Samarskiy et al., 2009) for calculating model coefficients. In terms of information, these variations can be considered as the system memory evolution (Grinchenko, 2004) changing the behavior of a system.

The dynamics of variables in such a flexible model fundamentally differs from the behavior of a classical model with a fixed structure. The model operation looks like a stream of classical models replacing one another similar to scenes in a movie.

It can be easily seen that neither the classical nor neoclassical modeling methodologies approach us to the required complexity and diversity of the behavior of natural systems, especially concerning the representation of living nature (Soukhovolsky, 2011).

In view of the aforesaid, the sacramental question arises: "What is to be done?" What are the prospects of solving the problems of planetary ecological and economic crisis and surviving on the planet, if we are not capable of predicting the behavior of complex systems because of the lack of effective modeling and prediction methods? Is there a way out the third – methodological global crisis?

The model self-assembly mathematics

It turns out that there is the way out. It consists in the creation of the mathematics of self-assembly and self-initiation of models, which could transform a system of an arbitrary number of equations to a model. The idea of such mathematics is grounded on the revelation of the fundamental properties of complex natural systems and the construction of the theory of self-organization, self-assembly of arbitrarily complex models on their basis. It should be noted that some of these properties have been already opened by modern science; their integration in a single, noncontradictory whole is just a problem of the knowledge sublimation to a certain level of theoretical generalizations.

Among fundamental world-outlook concepts broadening the current notions and forming the basis of the model self-generation mathematics is the idea that the stability of the observed environmental objects at all levels of their organization, from microcosm to cosmos, is grounded on the feedback mechanisms that effectively compensate the avalanche growth of chaos in very complex systems at their deviation from the current states of the dynamic equilibrium. Without highly organized matching of the interaction processes, the system either does not occur from many isolated components, falls to pieces just after the occurrence, or breaks from the previous stable state. Thus, we observe only the systems in which the feedback (Wiener, 1965) or adaptive mechanisms saved them from destructive environmental factors and fluctuation of their inner environment. When these mechanisms are ineffective, merely the transition forms with a limited lifetime arise.

Another fundamental discovery of the theory of adaptive self-organization is the fact that determinative chaos provides an effective tool for revealing the self-organization direction for

complex systems of any dimensionality. It is the main difference from the classical statistical concepts of the key role played by randomness in the multi-dimensional search, i.e. random quantity variation uncorrelated with the properties of a system.

The key aspect for keeping the stability of complex natural systems, which is reflected in the considered mathematics, is the presence of various stationary states, i.e. attractors or invariant diversities. Reconstruction of the dynamic equilibrium of a system is implemented by a fast stereotype response to destabilizing factors by the transition of the system to a certain attractor, which compensates the occurred instability. However, such transitions can occur only in the presence of the corresponding stationary states. When there are no such states, the system is randomized, which activates the subsystem of feedbacks to keep the system function by modifying the structure of the system. In other words, we observe the modification of the attractive landscape directed at the occurrence of the corresponding invariant diversities, which compensate the occurred instability threatening the existence of the system. The majority of invariant diversities reflecting the history of adaptation to the environmental variations form the characteristic invariant portrait of the system. The described processes occur within the limitations for ensembles of allowed trajectories where the viability and integrity of the system are safe.

The next important foundation of the mathematics of model self-initiation is complexity. In the first approximation, complexity of the system implies a sufficiently large set of elements and inter-element couplings. Sufficiency implies the possibility of constructing a hypersurface of the attractive landscape of a system that ensures its transition to one of the quasi-stationary states in response to almost any of destabilizing factors in the system surrounding. The set of elements and their interrelations can be reflected by a network (Strogatz, 2001) or, in the considered context, the self-organizing adaptive network, i.e. network with the stabilizing negative feedbacks, which is presented in nature by networks of interrelated atoms in molecules, cells in organisms, organisms in ecosystems, stars in galaxies, galaxies in the universe, and so on. Hierarchy of the element networks gives a fresh look at the universe, showing the integrity of the system organization at all levels of the universe supersystem. In this sense, we can explain its stability both in the whole and at all lower levels of the system organization. From the phenomenological point of view, the network structures can be brought into correlation with the finite, or infinite – in case of the universe, mathematical series of nonlinear basis functions, which allows synthesizing hypersurfaces of specified complexity. In such a formulation, complexity turns from the curse to the ally and provides to a researcher almost unlimited resources in combination with the acceptable power of computer systems.

The important peculiarity of the considered methodology is the emphasis on the informational side of the representation of reality. As was mentioned in study (Ecological Biophysics, 2002), the substance- and energy-based concepts have been deeply developed. However, the informational concept, in virtue of its complexity, remains underdeveloped. In context of the proposed system representations, information is presented not by a set of bits (Shannon, 1948) but by the combination of inter-element couplings in a network or system, which reflects the observed properties of the system.

It should be emphasized that the considered transformation of a cognitive model that became possible due to generalization of the recent scientific achievements returns us to realization of amazing simplicity, clearness, and harmony of the universe fundamentals opened in the past to founders of science and lost in the swelling chaos of numerous details and competing theories.

From concept to theory

The concept of adaptive self-organization of complex systems has been developed since the end of the 20th century (Lankin et al., 2012c). At the initial stages, it was called the concept of adaptive systems (Lankin et al., 2001). That time, the emphasis was the methods of neuroinformatics, whose algorithms were indented for adaptation of models with the theoretically unlimited dimensionality. The possibilities of neural networks are grounded by a number of theorems proved by different researchers. In particular, as was demonstrated in study (Gorban', 1998), the combination of summation, multiplication, and nonlinear transformation characteristic of the neural-network algorithms makes it possible to obtain any continuous function and, transferred to multidimensional images formed by neural networks, any continuous hypersurface. The theorem of equivalence of discrete networks to the Turing machine proved in (McCulloch, 1943) shows that the abilities of discrete networks are comparable with the abilities of computers.

During investigations, researches faced with the difficulties of using the classical neuroinformatics algorithms because the latter are specific and their application to modeling complex natural system is limited. To remove the limitations, it was proposed, first, to modify classical algorithms (Lankin et al., 1998), then, to change them for the algorithms based on the ideas of biological evolution (Lankin et al., 2004), and, finally, to use determinative algorithms of adaptive self-organization (Lankin et al., 2012a). The use of the latter allowed solving classical neuroinformatics problems, combining them with the classical models to reflect the described properties of complex natural systems (Lankin et al., 2012b), and transferring these problems to other types of the adaptive self-organization models (Lankin et al., 2012d).

The concept of adaptive self-organization offers a unique opportunity of building the models top-down, i.e., in a way natural for a man (Lankin et al., 2012c). In contrast to the classical methods, complex system properties that are of interest for the researcher are reproduced already at the first modeling stages. Only after that the structure is refined stepwise to a required depth with adaptation repeating for each new refinement level at controlling the preservation of the system properties of a model.

At the current stage of the development of the theory, the attention is focused on seeking the methods for refining the models of planetary processes and biosphere with its ecosystems (Lankin et al., 2012a,b,d,e) in order to find the ways of solving the problem of the global ecological crisis.

Population dynamics and Adaptive self-organization of complex systems

When this article was written and a journal to publish it was chosen, it appeared that the interdisciplinary topic of the article is strikingly consonant to the goals of the Journal of POPULATION DYNAMICS.

The search for generality in the descriptions of natural systems and phenomena belonging to different fields of knowledge has led the Journal's publishers to generalization of the concept of population as a group of biological organisms to sets of elements in physics, chemistry, etc. The general approach to construction of various models was demonstrated in study (Karev, 2012).

The concept of adaptive self-organization of complex systems, which was considered in this study, is also built on biological and ecological fundamentals. However, it was based on the systematic approach that allows extracting the desired generality of concepts already at the initial stages of the theory formation. This made it possible to pass easily from the proposed neural-network algorithms (Lankin et al., 2012a) describing the adaptive dynamics of the information model of the population of brain neurons to the ecological model of the dynamics of interactions in the stand populations (Lankin et al., 2012d).

In study (Ivanova et al., 2013) and previous works, the stand was modeled with the use of the catastrophe theory (Gilmor, 1993), which allows comparing the modeling methods. Different ways of searching for stationary regimes were proposed in studies (Nedorezov, 2013) and (Lankin et al., 2012d; Lankin et al., 2012b). Interesting is to compare the techniques of neural network application in population modeling that were reported in studies (Tonngang et al., 2012) and (Lankin et al., 2012b).

Thus, we see two approaches. One of them is the search for generality in many existing methods for population dynamics modeling; the other is construction of new mathematical methods on the basis of the revealed generality in analogous processes of various natural systems consisting of interacting populations. Generally, these two interdisciplinary approaches successfully supplement each other.

Conclusion

Suppose, behind the rows of this text the attentive reader has noticed the new world of boundless opportunities that opens beyond the customary ideas dictated by the ruling paradigm. The way from the known and usual to the new and unexplored is always hard. However, the thoughts expressed here suggest the parallel with the industrial revolution, which led to the change of manual, hand-made production of exclusive things for wholesale manufacture of high-quality, cheap goods. Physical labor gained new, unusual forms and exponentially transformed the society and the Earth's nature, which were almost invariable for thousands and thousands years. The concept of model self-initiation brings us to the new frontiers of evolution of the human mind, which is given a unique instrument of self-generation of new ideas and predictions with minimum participation of the creators. The pass from the industrial revolution of the force to the industrial revolution of the mind makes us face new questions transforming the considered ideas from the intellectual to moral sphere. The way we choose to transfer from the biosphere to the noosphere (Vernadsky, 2007) will resolve if we have the future or not.

Acknowledgements

The author thanks Prof. R.G. Khlebopros for his original view on the theory of adaptive self-organization reflected here in the model self-initiation concept and Prof. Yu.L. Gurevich, V.G. Sukhovol'skii, S.V. Khizhnyak, and N.S. Pechurkin for useful discussions.

References

- Anischenko V.S. 2008. Introduction to nonlinear dynamics. Moscow: LKI.
- Bak P. 1996. How Nature Works. The Science of Self-Organized Criticality. New York: Springer-Verlag.
- Crownover R.M. 1995. Introductions to Fractals and Chaos. Boston, London: Jones and Bartlett Publishers.
- Ecological Biophysics. 2002. Ecology and Biophysics: Time for Integration/ Ed. by Academician I.I. Gitel'zon, Prof. N.S. Pechurkin. Moscow: Logos. 3(3).

- Gilmore R. 1993. Catastrophe Theory for Scientists and Engineers. N.Y.: Dover Publications Inc..
- Gorban' A.N. 1998. Generalized approximation theorem and computational abilities of neural networks// *Sibirskii zhurnal vychislitel'noi matematiki*. 1(1): 11-24
- Grinchenko S.N. 2004. System Memory of Living Nature as a Basis of Its Metaevolution and Periodic Structure. Moscow: Mir.
- Haken H. 2010. Information and Self-Organization: A Macroscopic Approach to Complex Systems. Berlin: Springer.
- Haykin S. 1998. Neural Networks: A Comprehensive Foundation. New Jersey: Prentice Hall.
- Ivanova N.S., Zolotova E.S. 2013. Model of forest restoration// *Population Dynamics: Analysis, Modelling, Forecast* 2(2): 50-60.
- Karev G.P. 2012. The HKV method of solving of replicator equations and models of biological populations and communities// *Pop. Dyn.: Analysis, Modelling, Forecast*, 1(1): 1-31.
- Krapivin V.F., Potapov I.I. 2002. Methods of ecoinformatics. Moscow: VINITI RAN.
- Lankin Yu.P., Baskanova T.F. 2004. Algorithms of self-adaptation for atmospheric model designing// *SPIE*. 5397: 260-270.
- Lankin Yu.P., Baskanova T.F., Lobova T.I. 2012a. Neural-network analysis of complex ecological data// *Modern problems of education and science (Sovremennye problemy nauki i obrazovaniya)*. 4. URL: www.science-education.ru/104-6754.
- Lankin Yu.P., Baskanova T.F., Pechurkin N.S. 2012b. Modeling of adaptive self-organization of ecosystems// *Modern problems of education and science (Sovremennye problemy nauki i obrazovaniya)*. 5. URL: www.science-education.ru/105-6735.
- Lankin Yu.P., Ivanova N.S. 2012c. Fundamental paradigm of modeling the biosphere and its ecosystems// *Natural sciences*. 3(40): 96-104.
- Lankin Yu.P., Ivanova N.S., Baskanova T.F. 2012d. Fundamentals of the theory of modeling the biosphere and its ecosystems// *Natural sciences*. 3(40): 104-113.
- Lankin Yu.P., Khlebopros R.G. 1998. Self-adapting neural networks in solving ecological problems (possibility of searching behavior implementation)// *Environmental engineering (Inzhenernaya ekologiya)*. 4: 2-11.
- Lankin Yu.P., Khlebopros R.G. 2001. Ecological fundamentals of the concept of self-adapting networks and systems with the search behavior// *Environmental engineering (Inzhenernaya ekologiya)*. 2: 2-26.
- Lankin Yu.P., Mokogon D.A., Tereshin S.V. 2012e. Adaptive modeling of the planetary processes on the basis of satellite data// *Modern problems of education and science*

- (Sovremennye problemy nauki i obrazovaniya). 6. URL: <http://www.science-education.ru/106-7136>.
- Loskutov A.Yu., Mikhailov A.S. 2007. Bases of the theory of complex systems. Moscow-Izhevsk: NITc “Regulyarnaya i stokhasticheskaya dinamika”.
- Mandelbrot B.B. 2004. Fractals and Chaos. The Mandelbrot Set and Beyond. New York: Springer.
- Mantegna R.N., Stanley H.E. 2007. An introduction to economics: Correlation and complexity in finance. Cambridge: Cambridge University Press.
- McCulloch W., Pitts W. 1943. A logical calculus of the ideas imminent in nervous activity// Bull. Math. Biophys. 5: 115-137.
- Nedorezov L.V. 2013. Dynamics of Bupalus piniarius in Germany: search for stationary dynamical regimes// Population Dynamics: Analysis, Modelling, Forecast 2(2): 61–72.
- Pikovskiy A., Rozenblum M., Kurts U. 2003. Synchronization. The fundamental nonlinear phenomenon. Moscow: Tekhnosfera.
- Samarskiy A.A., Vabischevich P.N. 2009. Numerical methods of a solution of inverse problems of mathematical physics. Moscow: LKI.
- Schittkowski K. 1994. Parameter estimation in systems of nonlinear equations // Numerische Mathematik. 68: 129-142.
- Shannon C.E. 1948. A Mathematical Theory of Communication // Bell System Technical Journal. 27: 379-423.
- Soukhovolsky V.G. 2011. Modeling of ecological systems: problems and possible solutions // Materials of conference “Mathematical modeling in ecology” EkoMatMod-2011. Puschino: IFHiBPP RAN: 259-261.
- Strogatz S.H. 2001. Exploring complex networks// Nature. 410: 268-276.
- Tonnang H.E.Z., Löhr B., Nedorezov L.V. 2012. Theoretical study of the effects of rainfall on the population abundance of Diamondback moth, *Plutella xylostella*// Population Dynamics: Analysis, Modelling, Forecast 1(1): 32-46.
- Vernadsky V.I. 2007. Biosphere and Noosphere. Moscow: Airis-press.
- Wiener N. 1965. Cybernetics, on Control and Communication in the Animal and the Machine. New York: The MIT Press.
- Wiggins S. 1997. Introduction to Applied Nonlinear Dynamical Systems and Chaos. New York: Springer-Verlag.