Model of forest restoration

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Abstract

For Zauralsky (Trans-Ural) hilly piedmont province (Middle Ural) restoration of forest vegetation after clear-cuttings to forest stands of 80-160 years old was studied. Two options of the drained habitats have been studied: steep slopes of the southern exposition with small stony soils and the lower parts of gentle slopes with thick soils. Complex researches have been conducted. Restoration of the wood and grass- dwarf shrub layers has been studied. Biomass in absolutely dry condition was used as the integrated characteristic of the role of plants in the community. For the description of density dynamics (biomass) of a pines (*Pinus sylvestris* L.) and a birches (*Betula pendula* Roth, *B. pubescens* Ehrh.), their interference in the course of formation of the forest stand were constructed on the basis of systems the connected logistic equations of the model of their coexistence. Within the mathematical theory of catastrophes of René Tom the nonlinear model of formation of the tree layer structure on clear-cuttings depending on two managing parameters, such as thickness of soil (soil depth) and the intensity of the development of the grass- dwarf shrub layer. The satisfactory agreement between theory and experimental data was revealed. Objective quantitative methods for assessing the sustainability of forest restoration have been proposed. They are based on the definition of the type of the potential function, the calculation of their susceptibility and the distance from the separatrix position.

Key words: forest ecosystem, restoration and age dynamics, change of tree species, soil depth (capacities of soils), grass layer, mathematical modelling, catastrophe theory, forecast

Introduction

The quantitative prediction of restoration and age dynamics of forests is the central problem within the conception of the stable management of the resources. Forest ecosystems complexity, duration, polyvariation and non-linearity of the dynamic processes proceeding in them is reason of the absence of authentic generally accepted methods of the analysis of digressive-demutational changing (Lankin, Ivanova, 2012). In connection with this, the development of theoretico-methodological and methodical bases of the diagnostics of crisis processes and quantitative prognostification of forest dynamics is of current importance. The main factors of forest reduction in Russia are clear-cuttings and fires (Sannikov, 1992). The problem of forest restoration in the place of clear-cuttings and cuttings-burning is widely discussed in the literature. A number of hypotheses concerning mechanisms of succession changing has been suggested (Connell, Slatyer, 1977; Sannikov, 1992; Frelich, Reich, 1999). It is possible to derive quantitative forecasts of the forest ecosystem dynamics on their basis. In order to make quantitative forecasts of the formation of forest vegetation in the cuttings, the models on the base of the systems concerned with differential logistic equations (Ivanova, 2009), matrix models of competing structured populations of species-dominants (Ulanova, Zavalishin, Logofet, 2007; Logofet, 2013), as well as complex imitation models (Kellomaki et all., 1992; Meen, Nielsen, Ohlson, 2012) are being developed. Undeservedly little attention is being paid to the theory of catastrophes of R. Thom (Thom, Zeeman, 1975) though there are examples of its successful utilization and a number of interesting conclusions has been obtained on its base (Jones, 1977; Wright, 1987; Frelich, Reich, 1999; Isaev et all., 2008; Guts, Volodchenkova, 2012).

The aim of our research is the construction, within the framework of the theory of catastrophes, of the quantitative mathematical model of forest vegetation formation afterclear-cuttings depending on two controlling parameters: intensity of grass-dwarf shrub layer development and the conditions of forest growth (soil thickness), its verification on the base of the experimental data. The main task is to develop objective quantitative methods for the appreciation of the probability of dominant-forming species changing.

Material and methods

The area of the research is situated in the Zauralsky (Trans-Ural) hilly piedmont province (Middle Ural, Russia) between 57°00´–57°05´ N.lat. and 60°15´–60°25´ E.l. It is divided foothills formed by the alternation of meridian heights and ridges (Kolesnikov et all., 1974). Absolute heights are 200-500 m above sea-level. The climate is temperately cold, temperately damp. Frostless period is 90-115 day, average annual temperature is $+1^0$C (Kolesnikov et all., 1974).

We have studied two variants of drained places of residence:

1. Steep slopes of the southern exposition with small stony soils and very unstable water conditions. Cowberry shrub pine forests are the natural forest type (fig. 1).

2. The lower parts of gentle slopes with thick soils (more than 50 cm) and stable water conditions. Grass pine forests grow here (fig. 2).

We described earlier, these forests (Ivanova, Zolotova, 2013).

Fig. 1 Cowberry shrub pine forests: a – general view, b – soil

Fig. 2 Grass pine forests: a – general view, b – soil

Restoration age dynamics from clear-cuttings (4-5 years old) up to tree stands of 65-160 years old has been studied for these places of residence. In the steep slopes of the southern exposition with small stony soils restoration proceeds without changing tree species. Long-produced grass-reed grass birch forests are formed in the lower parts of gentle slopes with drained soils. The study of tree vegetation in the experimental areas has been carried out according to generally accepted methods (Iziumsky, 1972). Biomass in absolutely dry condition has been used as the integral characteristic of the phytocenotical role of plants. In order to determine the productivity of grass-dwarf shrub layer from 7 to 20 record plots of 1x1 m were established. The mass of forest forming tree species was determined by the way of calculation (Iziumsky, 1972; Usoltsev, 1997).

In order to describe the dynamics of density (biomass) of a pines (*Pinus sylvestris* L.) and a birches (*Betula pendula* Roth, *B. pubescens* Ehrh.), their mutual influence in the process of tree stand formation we constructed models of their joint existence on the base of the systems of connected logistic equations. We used the following system of differential equations (Lotka-Volterra model; Lotka, 1925; Bazykin, 1985):

$$\frac{dx_1}{dt} = A_1 x_1 - B_1 x_1^{\,2} + C_1 x_1 x_2$$

$$\frac{dx_2}{dt} = A_2 x_2 - B_2 x_2^{\,2} + C_2 x_1 x_2 \qquad (1)$$

where x_1 is a density (overground mass, g/m^2 in absolutely dry condition) of a pines; x_2 is a density of a birches; A, B, C are parameters which are determined in the process of the solution of the inverse task. A quantitative description of the formation of stand structure after clear-cutting carried out in the catastrophe theory according GP Bystrai methods. (Bystrai, Ivanova, 2010; Ivanova, Bystrai, 2010). We evaluated the model parameters according to the residual functional:

$$F(t) = \sqrt{\sum_i (Yi(t) - Yi)^2},$$

where Yi are elements of sample, and Yi(t) are theoretical values.

Results and discussion

Temporal dynamics of forest vegetation formation after clear-cutting cowberry shrub pine forest is shown in Fig.3. The dynamics is studied up to 200 years old tree stand, beginning with 5 years old cuttings. Figure 3 shows clearly that pines predominate obviously by mass in all the studied interval of restoration-age changing. The difference in densities of the studied tree species is of 1-2 orders. However, the complete birch vanishing out of the structure of forming forests is not observed.

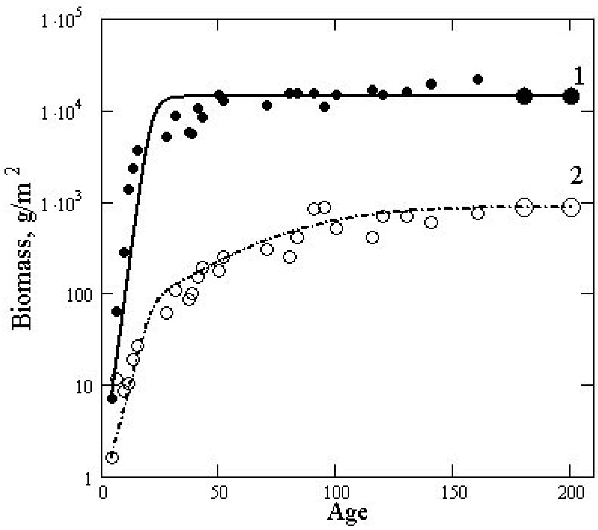

Fig. 3 Regeneration-age dynamics of density (biomass) of a pines (*Pinus sylvestris* L.) and a birches (*Betula pendula* Roth, *B. pubescens* Ehrh.) in the steep slopes of the southern exposition with small stony soils (10-15 cm): 1 – pines density (g/m^2), 2 – birches density (g/m^2), points – statistical data, lines – the results of the solution of the system of dependent non-linear logistic equations (1) (two last points in them – the prognosis for 20 and 40 years in advance). Equation coefficients: A_1 = 0.438; B_1 = 0.0000312; C_1 =0; A_2 = 0.231; B_2 = 0.0000438; C_2 = –0.0000138

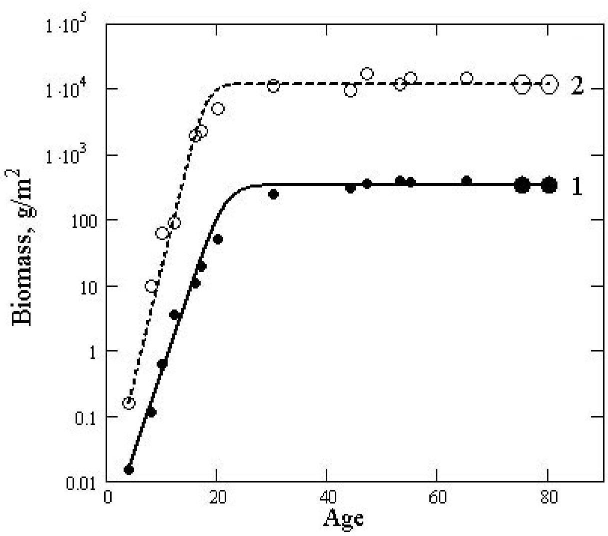

Fig. 4 Regeneration-age dynamics of density (biomass) of a pines and a birches in the lower parts of gentle slopes with thick (more than 50 cm) drained soils: 1 – pines density (g/m^2), 2 – birches density (g/m^2), points – statistical data, lines – the results of the solution of the system of dependent non-linear logistic equations (1) (two last points in them – the prognosis for 10 and 15 years in advance). Equation coefficients: A_1 = 0.575; B_1 = 0.0017; C_1 =0; A_2 = 0.805; B_2 = 0.000069; C_2 = 0

The time dynamics of forest vegetation formation after clear-cutting of grass pine forest in the lower parts of gentle slopes with thick (more than 50 cm) drained soils is shown in fig. 4. The dynamics is dealt with up to the tree stand of 80 years old, beginning from 5 years old cuttings.

Opposite correlation of tree species densities is observed in these forest growth conditions: birches prevail, pines are oppressed by them. Long derivative grass-red grass birch forests with strongly oppressed pines in the second layer are formed.

Thus, we have given mathematical description (on the base of the system of logistic equations) of two alternative ecodynamic series of forest vegetation formation in clear-cutting: regeneration of the initial forests (after cuttings in cowberry shrub pine forests) and the formation of long-derivative grass-reed grass birch forests (after cuttings in grass pine forests). Relying on these models and on the results stated in the previous papers (Bystrai, Ivanova, 2010; Ivanova, Bystrai, 2010) we will construct the model of tree layer formation in clear- cuttings depending on two controlling parameters:

$$\frac{d\rho}{dt} = -|k_1|\rho + |k_2|T\rho^2 - |k_3|\rho^3 + |k_4|H \qquad (2)$$

where k_i are parameters of the ecosystem which should be determined, p is the total biomass of pines and birches. H controlling parameters is the characteristic of the wealth of the forest growth conditions (thickness of soil, cm). T controlling parameter is dimensionless characteristics of the intensity of the grass layer development: $T=(p_0-p_h)/p_0$; p_h is a grass mass; $p_0 = (p_p + p_в)/2$ is the mean biomass of pines and birches; p_p , $p_в$ are pines and birch biomass accordingly.

The proposed model describes the effect of the forest growth conditions on the rate of tree plants growth and the oppression of forming tree vegetation by the grass layer and the less T (more grass biomass), the more tree vegetation is oppressed by grasses.

On the base of the experimental data which we have obtained, while solving the inverse task according to Byistrai's methodology (Bystrai, Ivanova, 2010), we determined values of all parameters of the equation (2), fig. 5. We evaluated the model parameters according to the residual functional. As a result, two equations for regeneration-age dynamics of total density of pines and birches have been obtained (fig. 5 a,b).

Equation (2) can be written in the canonical form (equation (3), i.e., without the quadratic term (Gilmore, 1984; Arnold, 1990). Eexample of the transition from the cubic equation $x^3 + \alpha x^2 + \beta x + \gamma = 0$ to a cubic equation $\eta^3 + a^*\eta + b^* = 0$ given in handbook (Korn, 1984) and in book (Bystrai, 2011). Formulas of this transition are: $x = \eta - \alpha/3$, $a^* = -\alpha^2/3 + \beta$, $b^* = 2(\alpha/3)^3 - \alpha\beta/3 + \gamma$.

$$\frac{d\eta}{dt} = -\left(\eta^3 + a^*\eta + b^*\right), \quad \frac{d\eta}{dt} = -\frac{\partial F^*}{\partial \eta}, \qquad (3)$$

$$F^*(\eta, a^*, b^*) = \frac{1}{4}\eta^4 + \frac{1}{2}a^*\eta^2 + b^*\eta, \qquad \eta = \rho^* - T\rho_0^*. \qquad (4)$$

Here $\eta = p_0/p_c - Tp_0/p_c$ is the parameter of the order characterizing the deviation of vegetation density (tree and grass) at the constant quantity of T, that is close to one, from some average meaning of pines and birches density of $p_0=(p_p + p_в)/2$; $\rho_0^* = |k_2|/3|k_3|\rho_c$; ρ_c is density scale; a^*, b^* are parameters: $a^*=-3(T^2p_0^*-1)$, $b^*=-H^*+3Tp_0^*-2T^3p_0^{*3}$. $b^*=-H^*+H_s^*$ can be represented as a sum of the external field of H^* and internal self-consistent $H_s^*=3Tp_0^*-2T^3p_0^{*3}$. $H^*=H/H_c$, H_c is critical soil thickness. When $b^*=0$, $H^*=H_s$. $F^*=F/F_0$ is the potential function of gathering catastrophe that defines energy characteristic in the cited form. For local stable states one of the minima is weak. This corresponds to a metastable state. Potential function for studied ecodynamic series are shown in fig. 6. System state stability is determined by their form. Availability of the local and global minimum in general case is determined with Thom theorem, in the case of gathering catastrophe – with non-zero values of the b^* (at $b^*=0$ potential is symmetric). For two particular situations which we have observed, b^* is different from 0: for cowberry shrub pine forests $b^*=-1.54*10^8$; for birch forests $b^*=-2.22*10^5$. The difference between the examined pine forests and birch forests, by b^* parameter makes up 3 orders.

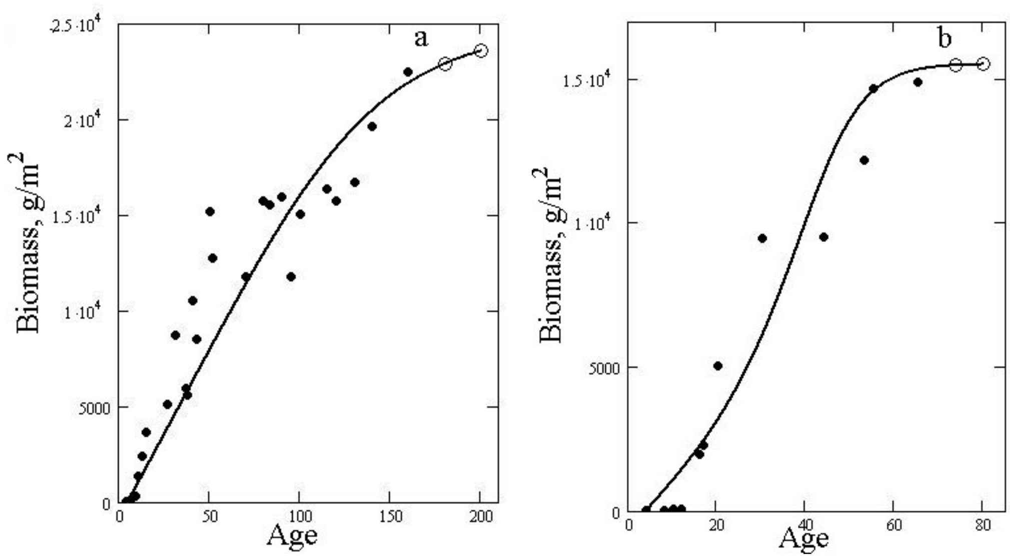

Fig. 5 Regeneration-age dynamics of the summary density of pines and birches (points – empirical data, lines – the results of the equation solution (1)): a – cowberry shrub pine forests in the steep slopes of the southern exposition with small stony soils (H = 10–15 cm) (two last points in the line – the prognosis for 9 and 15 years); equation coefficients: $k_1=8.33*10^{-8}$, $k_2T=1.25*10^{-7}$, $k_3=1.67*10^{-11}$, $k_4H=175$; b – grass birch forests in the lower parts of gentle slopes with thick (H is more than 50 cm) drained soils (two last points in the line – the prognosis for 5 and 10 years in advance); equation coefficients: $k_1=9.17*10^{-6}$, $k_2T=8.33*10^{-6}$, $k_3=5.83*10^{-10}$, $k_4H=175$.

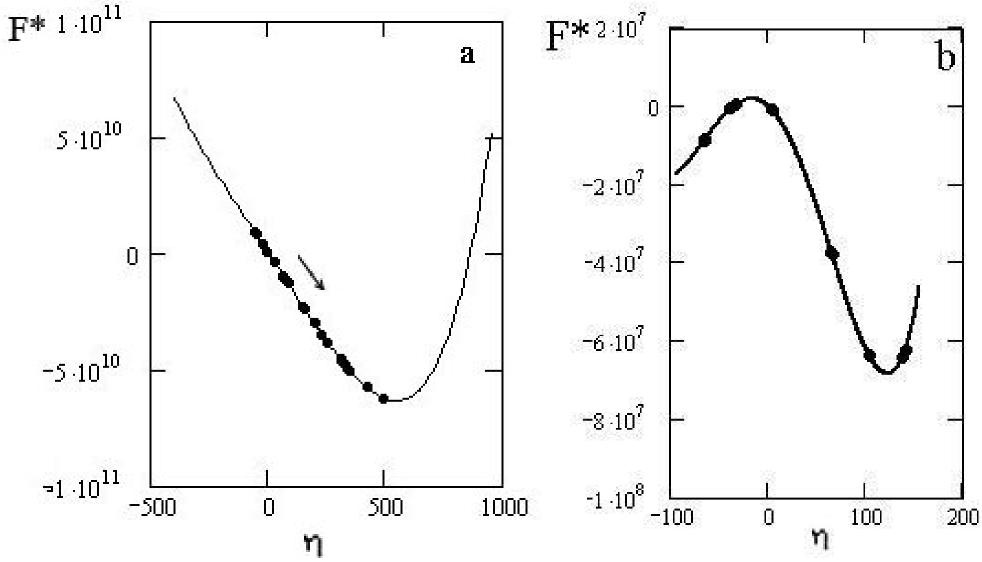

Fig. 6 Potential functions (points – empirical data, lines – the result of the equation solution (4)): a – cowberry shrub pine forests in the steep slopes of the southern exposition with small stony soils; б –grass birch forests in the lower parts of gentle slopes with thick drained soils

By Thom' theorem for disaster assembly the following special (in the mathematical sense) points were introduced.

1. $\frac{dF^*}{d\eta} = 0$ $\eta^3 + a^*\eta + b^* = 0$ - degenerate points (corresponding to the extreme of the potential function F);

2. $\frac{d^2F^*}{d\eta^2} = 0$ $3\eta^2 + a^* = 0$ - doubly degenerate points along the lines BG, SG (solutions corresponding to the two extremes of the potential functions are equal);

3. $\frac{d^3F^*}{d\eta^3} = 0$ $6\eta = 0$ - thrice degenerate point G (solutions corresponding to the three extremes of the potential function are equal to 0). Separatrix equation follows from the joint solutions 1 and 2 (fig. 7).

We can judge the stability of the structure of the forest (the probability of changing edificator) based on the calculation of the distance from the separatrix. The equation of the separatrix of forest vegetation $(a^*/3)^3+(b^*/2)^2= 0$ characterize a limit to maintain dominance woody species (pine or birch) in the stand.

For cowberry shrub pine forests which we have studied $(a^*/3)^3+(b^*/2)^2=5.92*10^{15}$, and for grass-reed grass birch forests $(a^*/3)^3+(b^*/2)^2=-6.88*10^{10}$. These meanings show sufficient remoteness from separatrix both cowberry shrub pine forests and grass-reed grass birch forests. This is the evidence of the fact that the edificators change is unlikely.

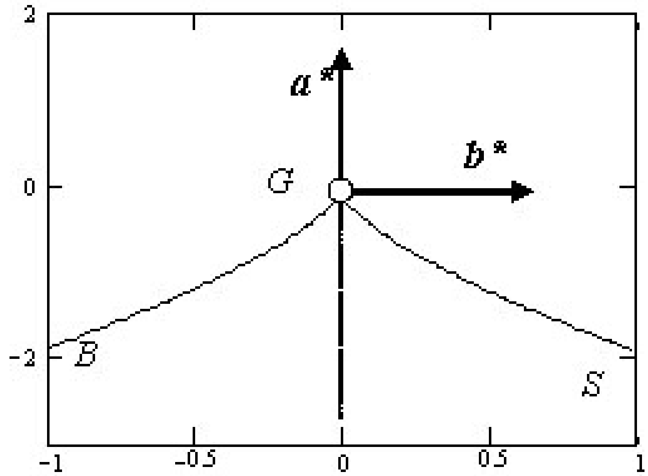

Fig. **7** BGS separatrix developing forest vegetation

The approach which we have suggested allows calculation susceptibility for the equation of $\eta^3 + a^*\eta + H_s^* = H^*$ (characteristic of the change of variable at the change of *H** external field):

$$\chi = \frac{\partial\eta}{\partial H^*} = \frac{1}{3\eta^2 + a^*} = -\frac{1}{2a^*} = \frac{1}{6(T^2\rho_0^{*2} - 1)}.$$

Under approaching the critical point $a^* = b^* = \eta \to 0$, and the susceptibility tends to infinity. For cowberry shrub pine forests which we study the susceptibility amounts to $4.46*10^{-5}$, and grass-reed grass birch forests – 0.038.

Thus, the existence of alternative lines of succession dynamics (pine forests and birch forests) has been formalized mathematical by means of the catastrophe theory. The results are represented in the visual form: in the form of the plots of potential functions. Objective quantitative methods of the estimation of stability of regeneration-age dynamics providing well-founded predicting of the condition of described objects were developed. As the main criteria of the stability of the development we suggest: remoteness from separatrix and the value of susceptibility.

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References

Arnold, V.I. 1990. Catastrophe Theory. Nauka.

Bazykin, A.P. 1985. Mathematical Biophysics of Interacting Populations. Moscow: Nauka.

Bystrai, G.P. 2011. The thermodynamics of irreversible processes in open systems. Moscow Izhevsk: SIC “Regular and Chaotic Dynamics”.

Bystrai, G.P. & Ivanova, N.S. 2010. Approaches to the Modelling of Forest Vegetation on the Base of the Theory of Catastrophes// Agrarian bulletin of Ural (Agrarnyj vestnik Urala). 2: 75–79.

Connell, J.H. & Slatyer, R.O. 1977. Mechanismus of succession in natural communities and their role in community stability and organization// Amer. Natur. 111 (982): 1119-1144.

Frelich, L.E. & Reich, P.B. 1999. Neighborhood Effects, Disturbance Severity, and Community Stability in Forests// Ecosystems. 2(2): 151–166.

Gilmore, R. 1984. Applied Theory of Catastrophes. Verlag, 1984. T.1., T.2.

Guts, A.K. & Volodchenkova, L.A. 2012. Cybernetics of Forest Ecocystem Catastrophes. Omsk: KAN Publ., 2012. 220 p.

Ivanova, N.S. 2009. Modelling of growth grass-shrub layer in the short- produced birch Southern Urals// Agrarian bulletin of Ural (Agrarnyj vestnik Urala). 4: 96–98.

Ivanova, N.S. & Bystrai, G.P. 2010. Model of the formation of the tree layer structure on cuttings. Part 1. The control parameters// Agrarian bulletin of Ural (Agrarnyj vestnik Urala). 5: 85–89.

Ivanova, N.S. & Zolotova, E.S. 2013. Biodiversity of the natural forests in the Zauralsky hilly piedmont province// Modern problems of education and science (Sovremennye problemy nauki i obrazovanija). 1. URL: http://www.science-education.ru/107-8563

Isaev, A.S., Soukhovolsky, V.G., Buzykin, A.I. & Ovchinnikova T.M. 2008. Succession processes in forest communities: models of phase transitions// Conifers of the boreal area. 1-2: 9-15.

Iziumsky, P.P. 1972. Valuation of the thin wood. Forest industry. Moscow. 88 p.

Jones, D.D. 1977. Catastrophe theory applied to ecological systems// Simulation. 29 (1): 1–15.

Kellomaki, S., Vaisanen, H., Hanninen, H. Kolstrom, T. Lauhanen, R. Mattila, U. & Pajari, B. 1992. A simulation model for the succession of the boreal forest ecosystem// Silva Fennica. 26: 1–18.

Kolesnikov, B.P., Zubareva, R.S. & Smolonogov, E.P. 1974. Forest vegetation conditions and forest types of the Sverdlovsk region. UNTS of Academy of science of the USSR. Sverdlovsk. 176 p.

Korn, G., Korn, T. 1984. Mathematical Handbook. Nauka.

Lankin, Yu.P. & Ivanova, N.S. 2012. Fundamental paradigm of modelling the biosphere and its ecosystems// Natural sciences. 3: 96–104.
URL: http://www.aspu.ru/images/File/Izdatelstvo/EN%203%2840%29%202012/96-104.pdf

Loehle, C. 1989. Catastrophe theory in ecology: a critical review and an example of the butterfly catastrophe// Ecol Modell. 49: 125–52.

Logofet, D.O. 2013. Projection matrix calibration under reproductive uncertainty: the maximization of λ_1 and the merit of indication// Population Dynamics: Analysis, Modelling, Forecast. 2(1): 1-22

Lotka, A.J. 1925. Elements of Physical Biology. Williams and Wilkins

Meen, E., Nielsen, A. & Ohlson, M. 2012. Forest stand modelling as a tool to predict performance of the understory herb *Cornus suecica*// Silva Fennica. 46(4): 479–499.

Miina, J., Hotanen, J.P. & Salo, K. 2009. Modelling the abundance and temporal variation in the production of bilberry (Vaccinium myrtillus L.) in Finnish mineral soil forests// Silva Fennica. 43(4): 577–593.

Sannikov, S.N. 1992. Ecology and geography of natural regeneration of *Pinus sylvestris*. Science. Moscow. 264 p.

Thom, R. & Zeeman, E.C. 1975. Catastrophe theory: its present state and future perspectives. Dynamical systems: Lecture notes in mathematics. 468 (4): 366-389.

Ulanova, N.G, Zavalishin, N.N. & Logofet, D.O. 2007. Competition between and within aspen (Populus tremula) and raspberry (Rubus idaeus) after clear-cutting: matrix models of structured populations dynamics// Forest Science and Technology. 3 (1): 68–77.

Usoltsev, V.A. 1997. Bioecological aspects of valuation of trees phytomass. Ural Branch of the Russian Academy of Sciences. Yekaterinburg. 216 p.

Wright, D.J. 1983. Catastrophe Theory in Management Forecasting and Decision Making// The Journal of the Operational Research Society. 34 (10): 935–942.

Zeeman, E.C. 1976. Catastrophe Theory// Sci Am. 234: 65–83.